Scientific seminar
SUSU Big Data and Machine Learning Lab

Parallel algorithm for imputation missing values of time series

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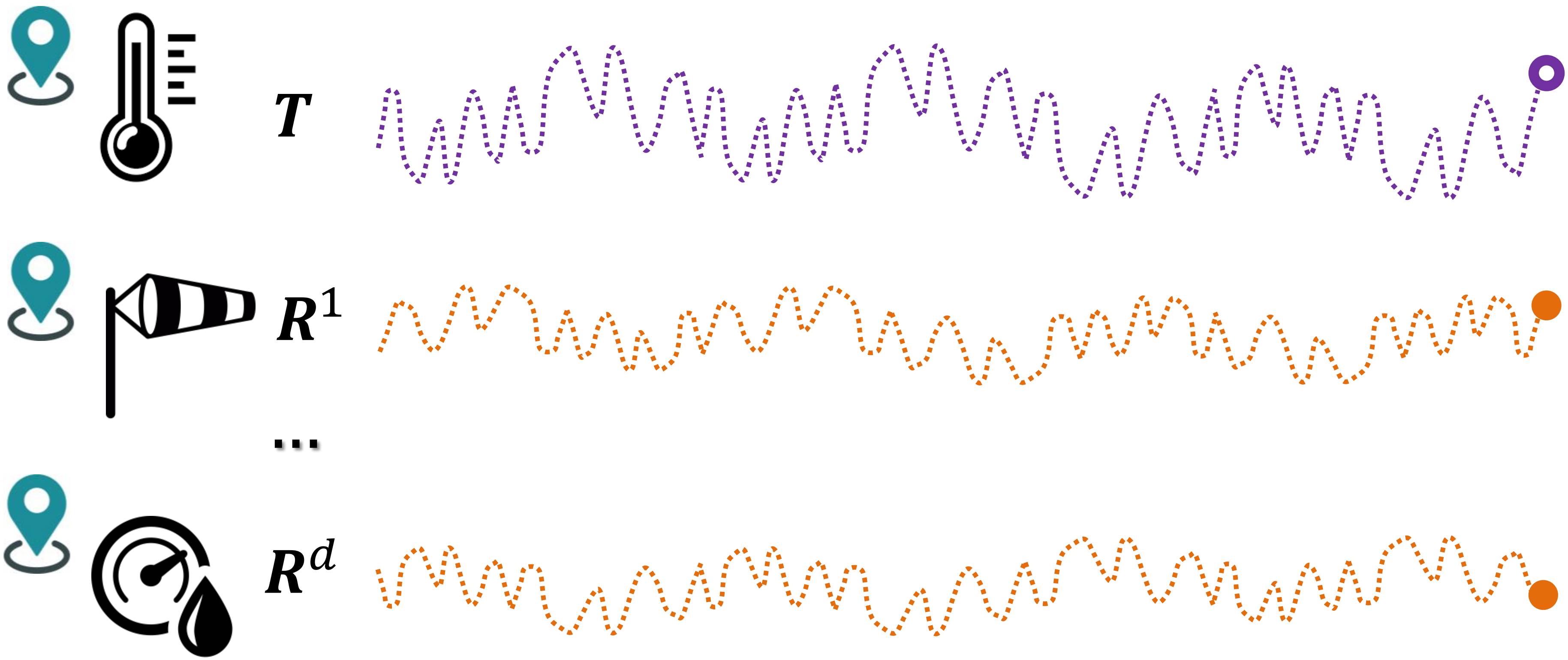
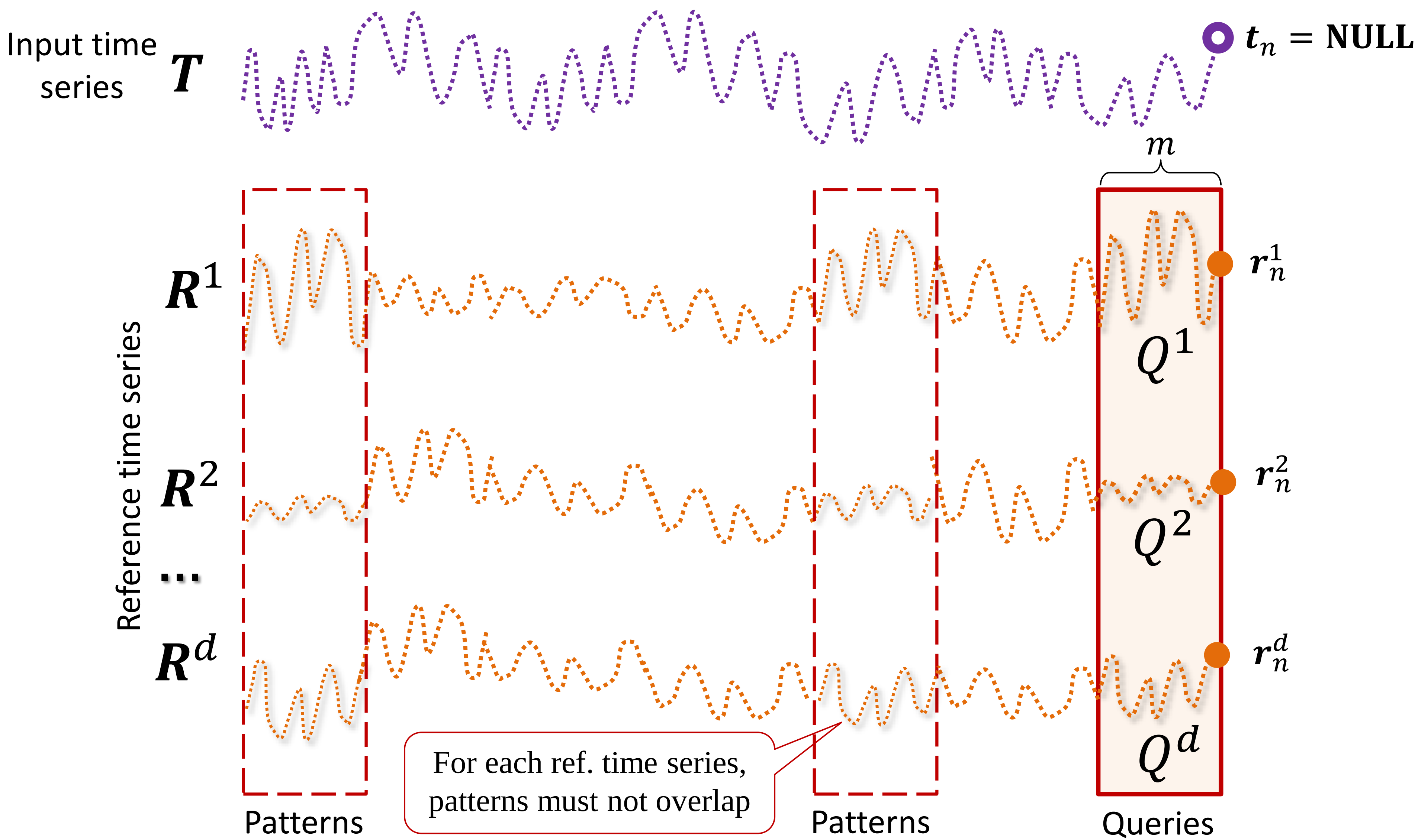
Imputation through reference time series



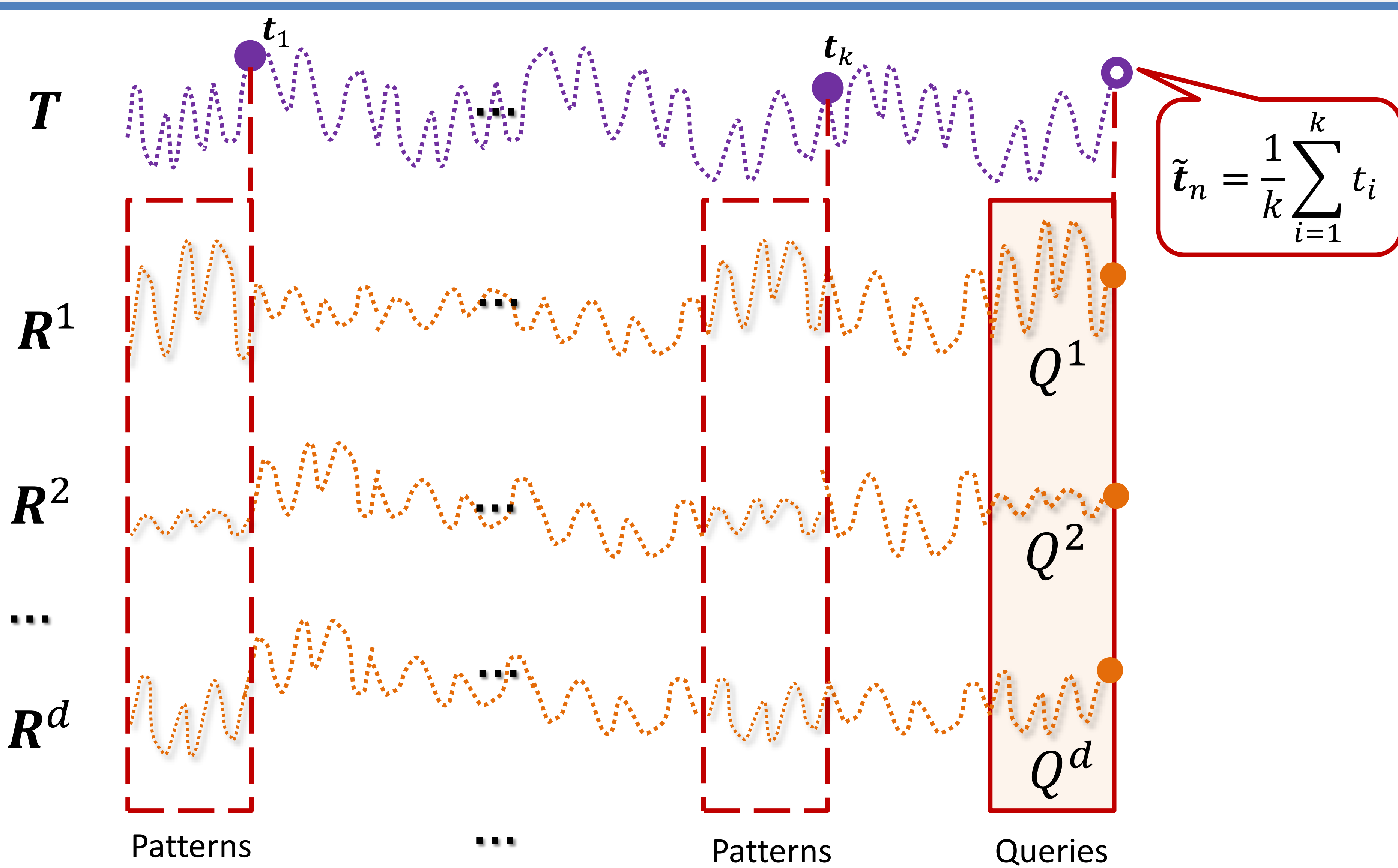
Heuristics

the time series undergo imputation and the reference time series behave similarly in the same subsequences

Imputation: Find k patterns



Imputation: Reconstruction of missing value



ParaDI: Parallel DTW-based Imputation*

- DTW (Dynamic Time Warping) distance measure
  – DTW is used in the pattern search, since it is the best to measure similarity of subsequences by their shapes
- Pruning through the lower bounds (LB)
  – LBs allow to discard clearly dissimilar subsequences without calculation of DTW
- Fork-join parallelism
  – OpenMP for x86, OpenACC for GPU

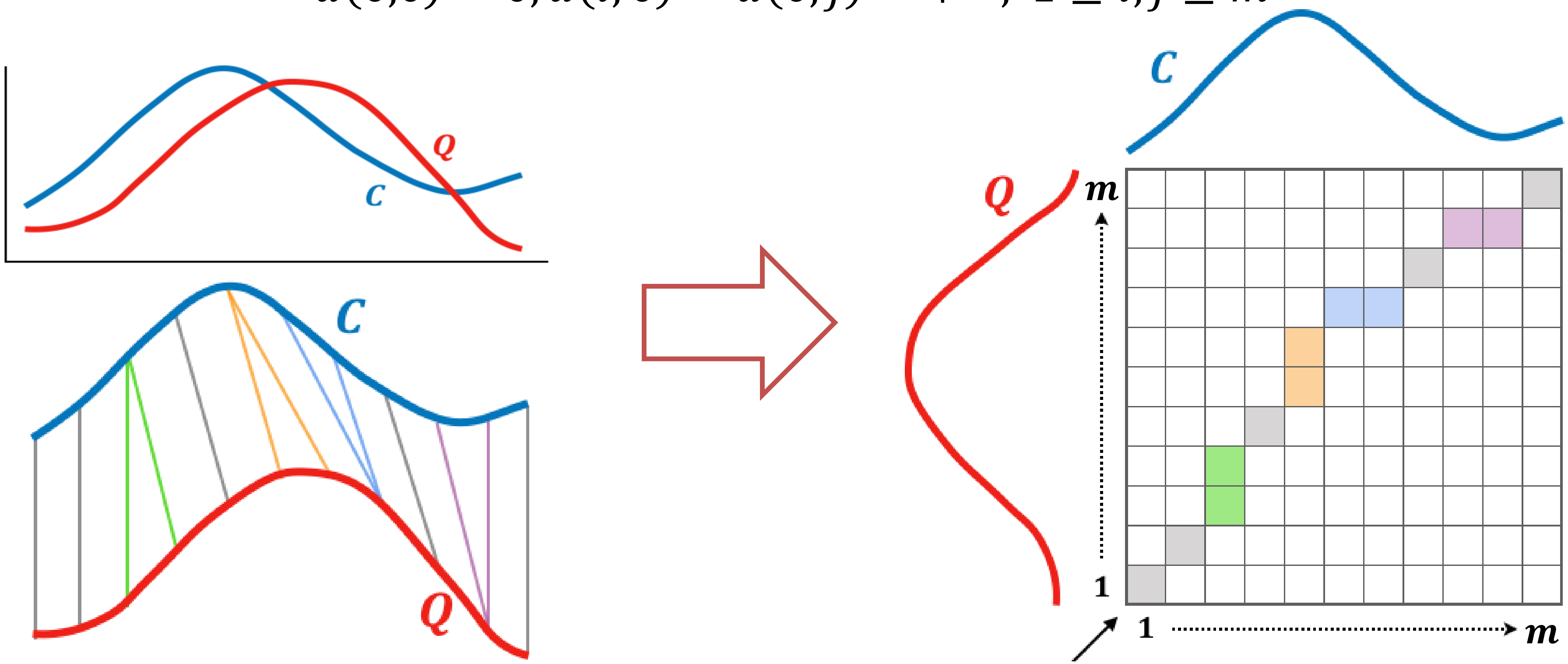
* Zymbler M.L., Poluyanov A.N., Kraeva Ya.A. Parallel Algorithm for Real-time Sensor Data Recovery for a Many-core Processor. Bulletin of the South Ural State University. Series: Computational Mathematics and Software Engineering. 2022. Vol. 11, no. 3. P. 68–89. (in Russian) DOI: 10.14529/cmse220305.

DTW (Dynamic Time Warping) distance measure*

$$\mathbf{DTW}(Q, C) = d(m, m)$$

$$d(i,j) = (q_i - c_i)^2 + \min \begin{cases} d(i-1, j) \\ d(i, j-1) \\ d(i-1, j-1) \end{cases}$$

$$d(0,0) = 0, d(i,0) = d(0,j) = +\infty;\ 1 \le i,j \le m$$



* Berndt D.J., Clifford J. Using Dynamic Time Warping to Find Patterns in Time Series. KDD & AAAI Workshop 1994. TR-WS-94-03. P. 359-370.

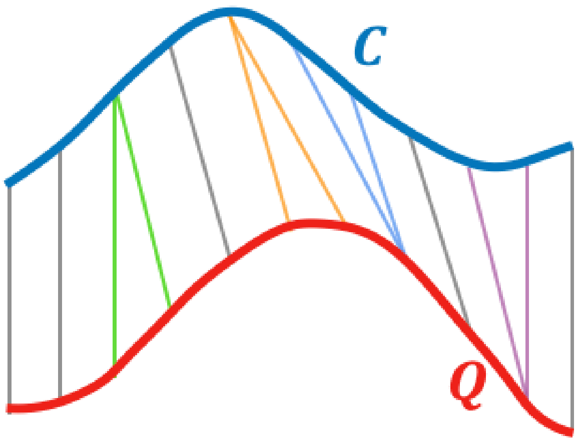
DTW: complexity *vs.* accuracy

Complexity
$\boldsymbol{O(m^2)}$

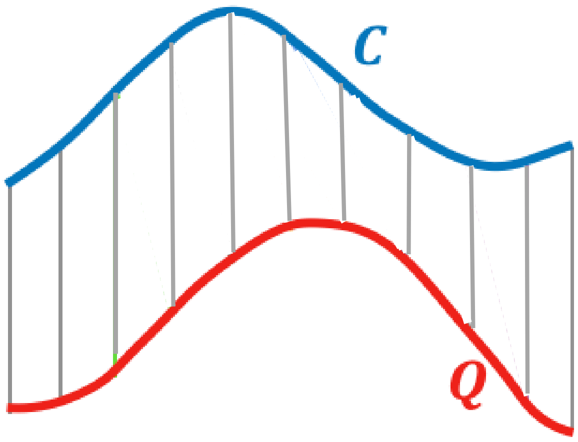
$$\mathbf{DTW}(Q, C) = d(m, m)$$

$$d(i,j) = (q_i - c_i)^2 + \min \begin{cases} d(i-1, j) \\ d(i, j-1) \\ d(i-1, j-1) \end{cases}$$

$$d(0,0) = 0, d(i,0) = d(0,j) = +\infty;\ 1 \le i,j \le m$$



DTW is better to measure the similarity by shape than the Euclidean distance



DTW acceleration: Sakoe–Chiba band*

Complexity
$\boldsymbol{O(mr)}$

$$\mathbf{DTW}(Q, C) = d(m, m)$$

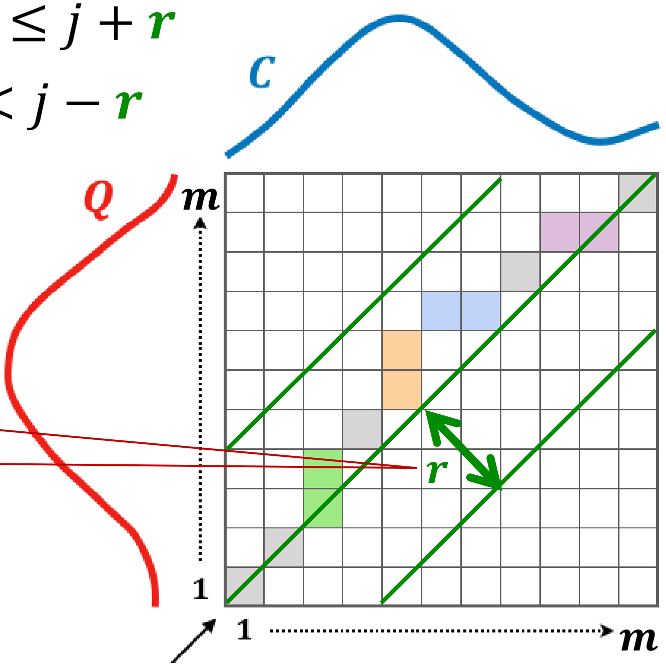
$$d(i,j) = (q_i - c_i)^2 + \min \begin{cases} d(i-1, j) \\ d(i, j-1) \\ d(i-1, j-1) \end{cases}$$

$$d(0,0) = 0, d(i,0) = d(0,j) = +\infty;\ 1 \le i, j \le m;$$

$$\mathbf{0} \le \boldsymbol{r} \le \boldsymbol{m} - \mathbf{1}, j - \boldsymbol{r} \le i \le j + \boldsymbol{r}$$

$$d(i,j) = +\infty, j + \boldsymbol{r} < i < j - \boldsymbol{r}$$

Complexity *vs.* accuracy trade-off



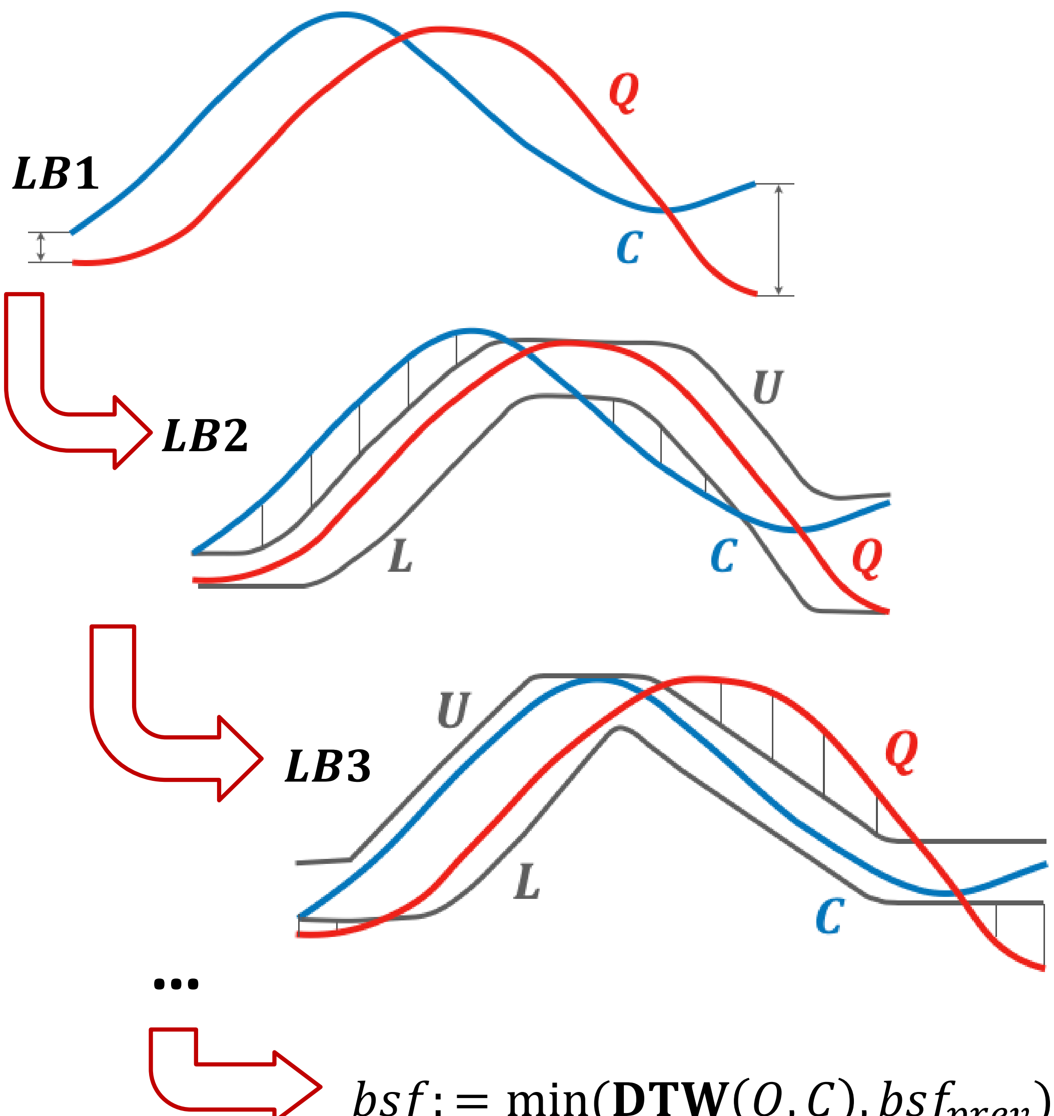
* Sakoe H., Chiba S. Dynamic Programming Algorithm Optimization for Spoken Word Recognition. IEEE Trans. on Acoustics, Speech, and Signal Processing. 1978. Vol. 26. P. 43-49.

DTW acceleration: Lower bounding*

- Lower bound function (LB)
  - $\mathrm{LB}: \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}_+$, complexity is lower than $O(m^2)$
    $\forall\, R[i{:}m], Q{:}\ \mathrm{LB}(R[i{:}m], Q) \leq \mathrm{DTW}(R[i{:}m], Q)$
- Lower bounding pruning
  - Best-so-far minimum of DTW: bsf
  - if $\mathrm{LB}(R[i{:}m], Q) > bsf$, then $\mathrm{DTW}(R[i{:}m], Q) > bsf$,
    so $R[i{:}m]$ is clearly dissimilar to Q,
    and we don't need to calculate DTW
  - It works only if $R[i{:}m]$ and Q are z-normalized
  - LBs can be applied in a cascade

* Rakthanmanon T., *et al.* Addressing Big Data Time Series: Mining Trillions of Time Series Subsequences Under Dynamic Time Warping. ACM Trans. Knowl. Discov. Data. 2013. Vol. 7, no. 3. 10:1–10:31.

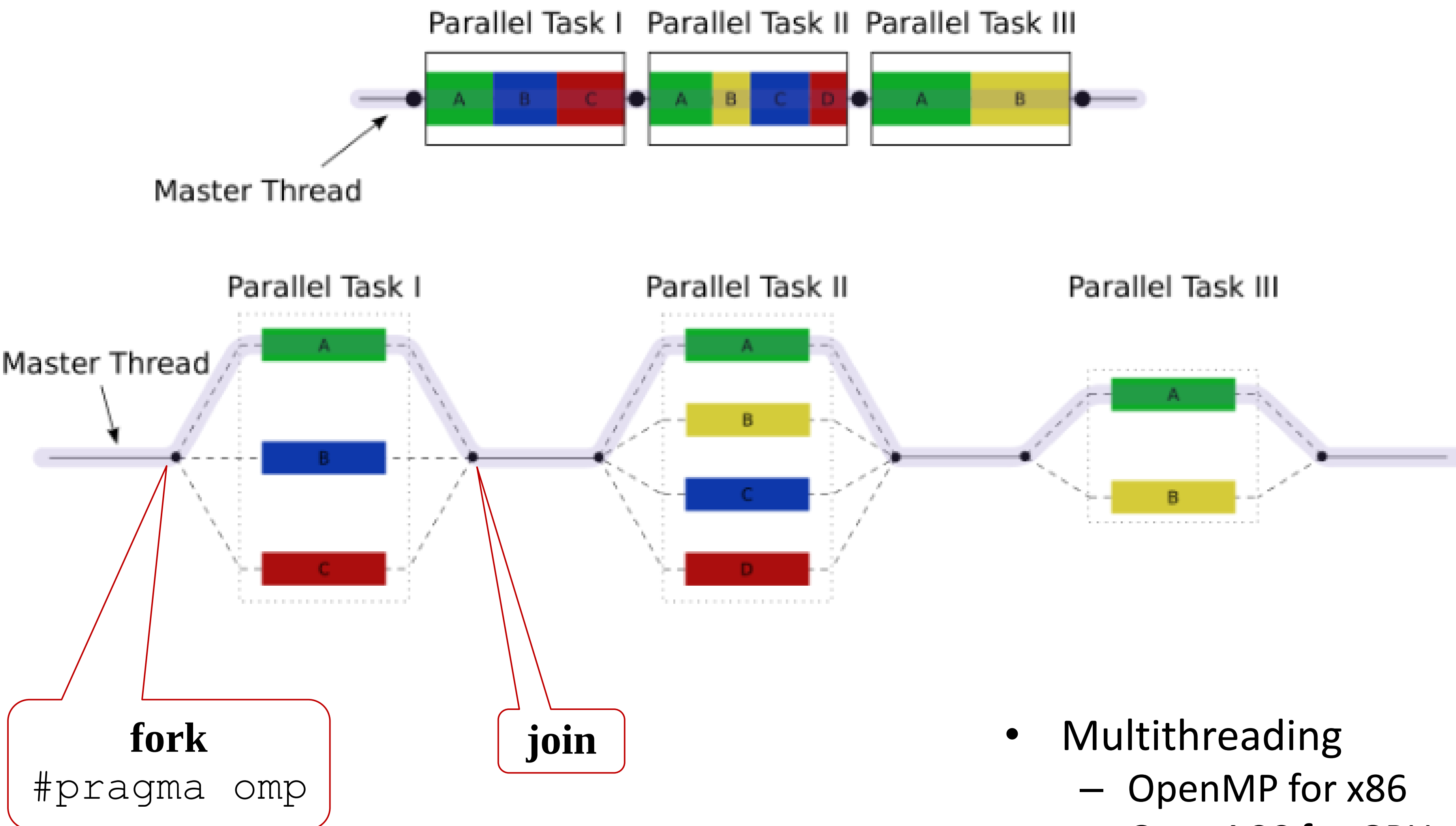
Lower bounding: Cascade of LBs



$bsf := \min(\mathbf{DTW}(Q,C), bsf_{prev})$

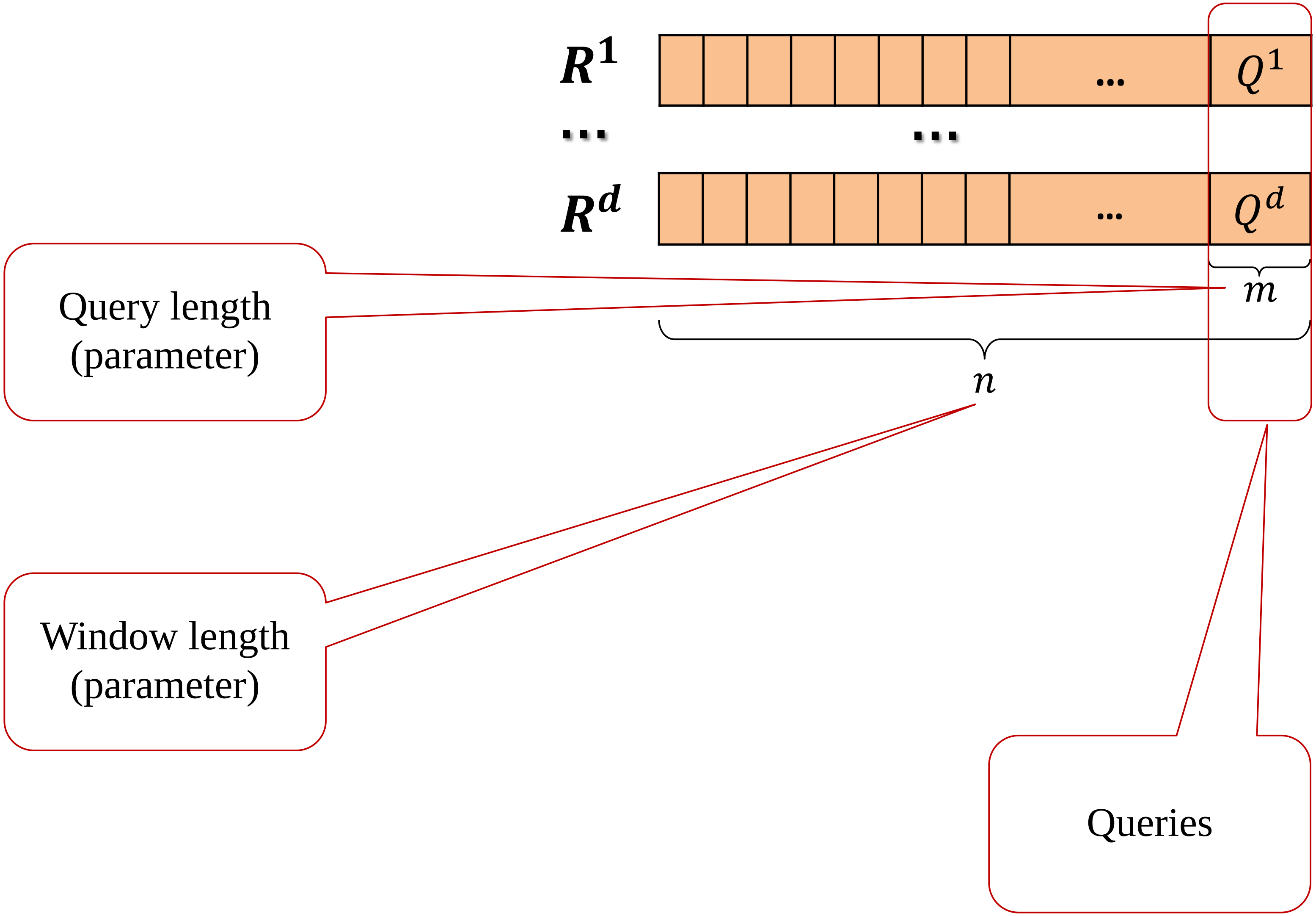
Lower bound	Complexity
LB_KimFL $\boldsymbol{LB1}(Q,C) = (q_1 - c_1)^2 + (q_m - c_m)^2$	$O(1)$
LB_KeoghEC $\boldsymbol{LB2}(Q,C) = \sum_{i=1}^{m} \begin{cases} (c_i - u_i)^2, & c_i > u_i \\ (c_i - \ell_i)^2, & c_i < \ell_i \\ 0, & otherwise \end{cases}$ $u_i = \max_{i-r \le k \le i+r} q_k$ $\ell_i = \min_{i-r \le k \le i+r} q_k$	$O(m)$
LB_KeoghEQ $\boldsymbol{LB3}(Q,C) = \boldsymbol{LB2}(C,Q)$	$O(m)$
LB_Yi, LB_PAA, LB_FTW, …	…
$\mathbf{DTW}(Q,C)$	$O(mr)$

Fork-join parallelism



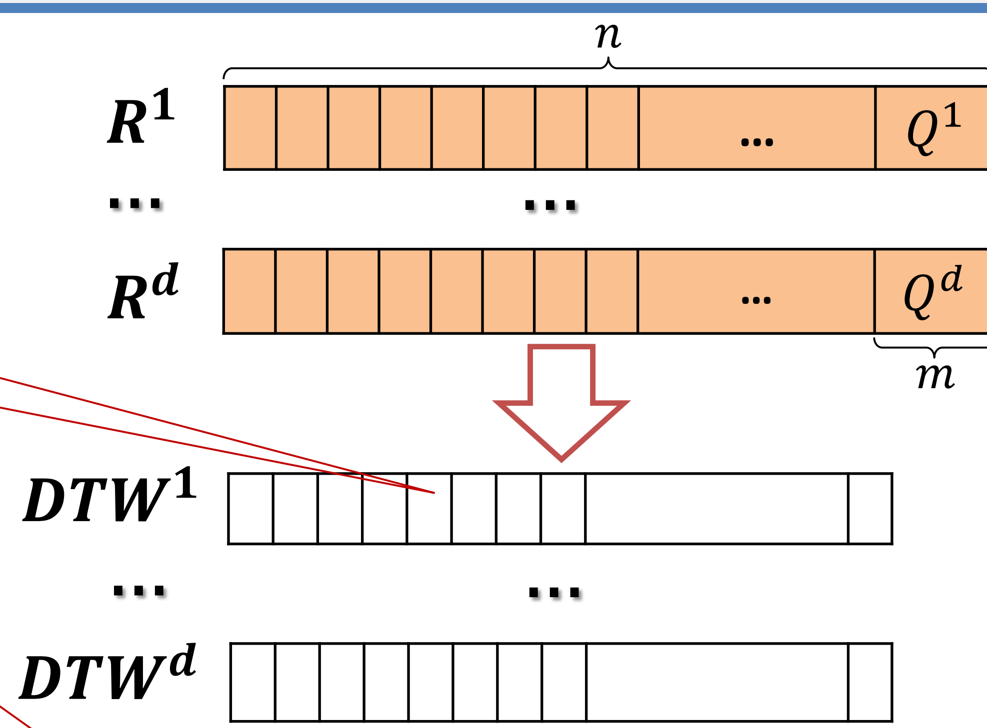
- Multithreading
  - OpenMP for x86
  - OpenACC for GPU

ParaDI: Reference time series



ParaDI: Pattern search

Distance between
the interval $R^i[j:m]$ and the query Q^i
w.r.t. the DTW distance measure



1. Pattern search

for $i := 1$ **to** d **do**

for $j := 1$ **to** $n - m + 1$ **do**

$DTW^i(R^i[j:m], Q^i)$

Green means **parallel through OpenMP**

ParaDI: Scoring

1. Pattern search

for $i := 1$ **to** d **do**

  for $j := 1$ **to** $n - m + 1$ **do**

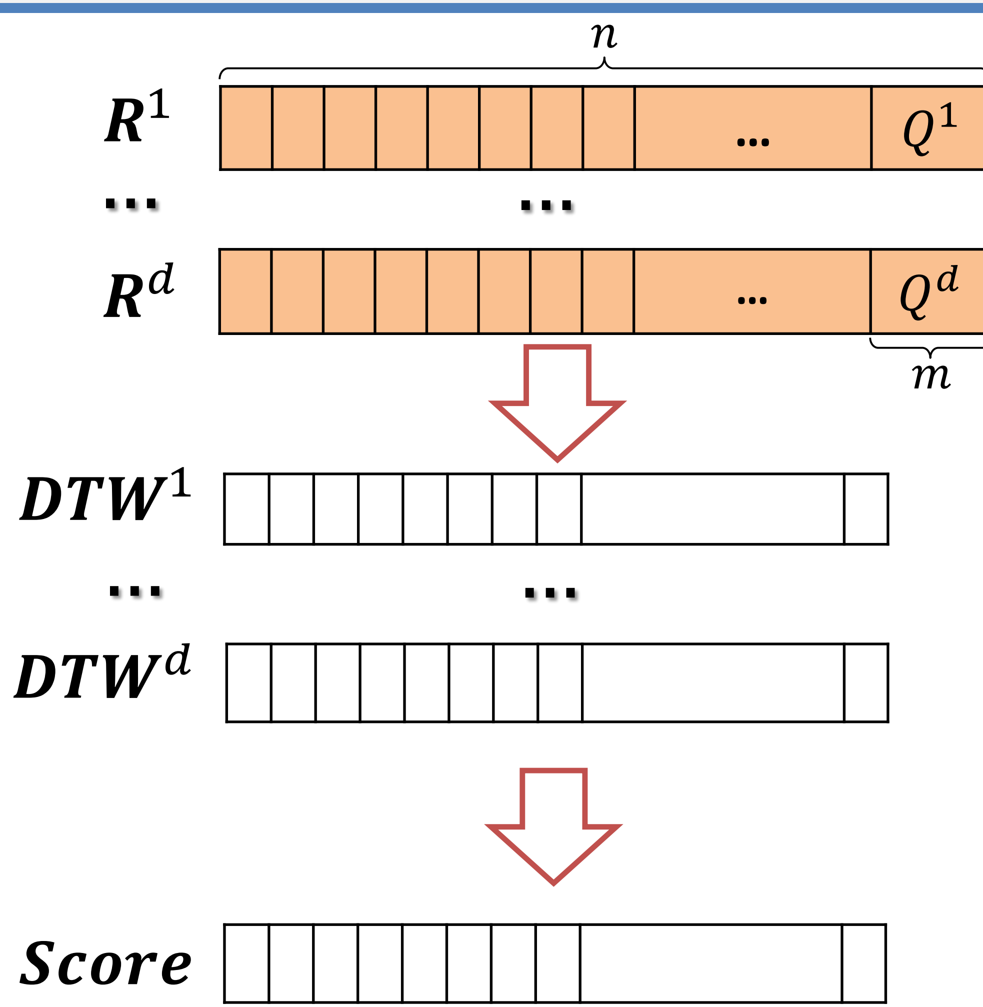
    $DTW^i(R^i[j:m], Q^i)$

2. Scoring

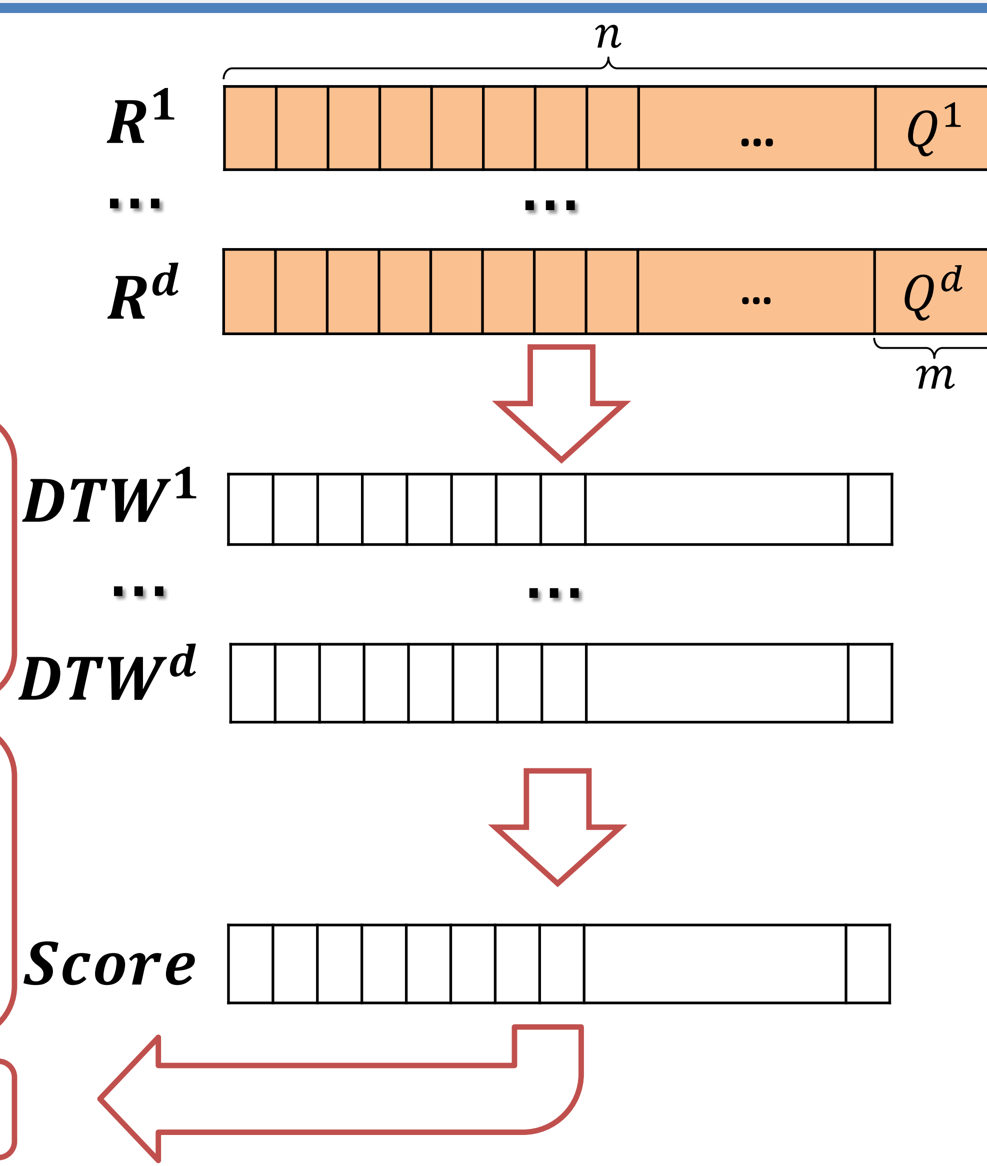
for $j := 1$ **to** $n - m + 1$ **do**

  for $i := 1$ **to** d **do**

    $Score(j) := Score(j) + \frac{weight(j,i)}{DTW^i(j)+\varepsilon}$



ParaDI: Reconstruction



1. Pattern search

for $i := 1$ **to** d **do**

for $j := 1$ **to** $n - m + 1$ **do**

$DTW^i(R^i[j:m], Q^i)$

2. Scoring

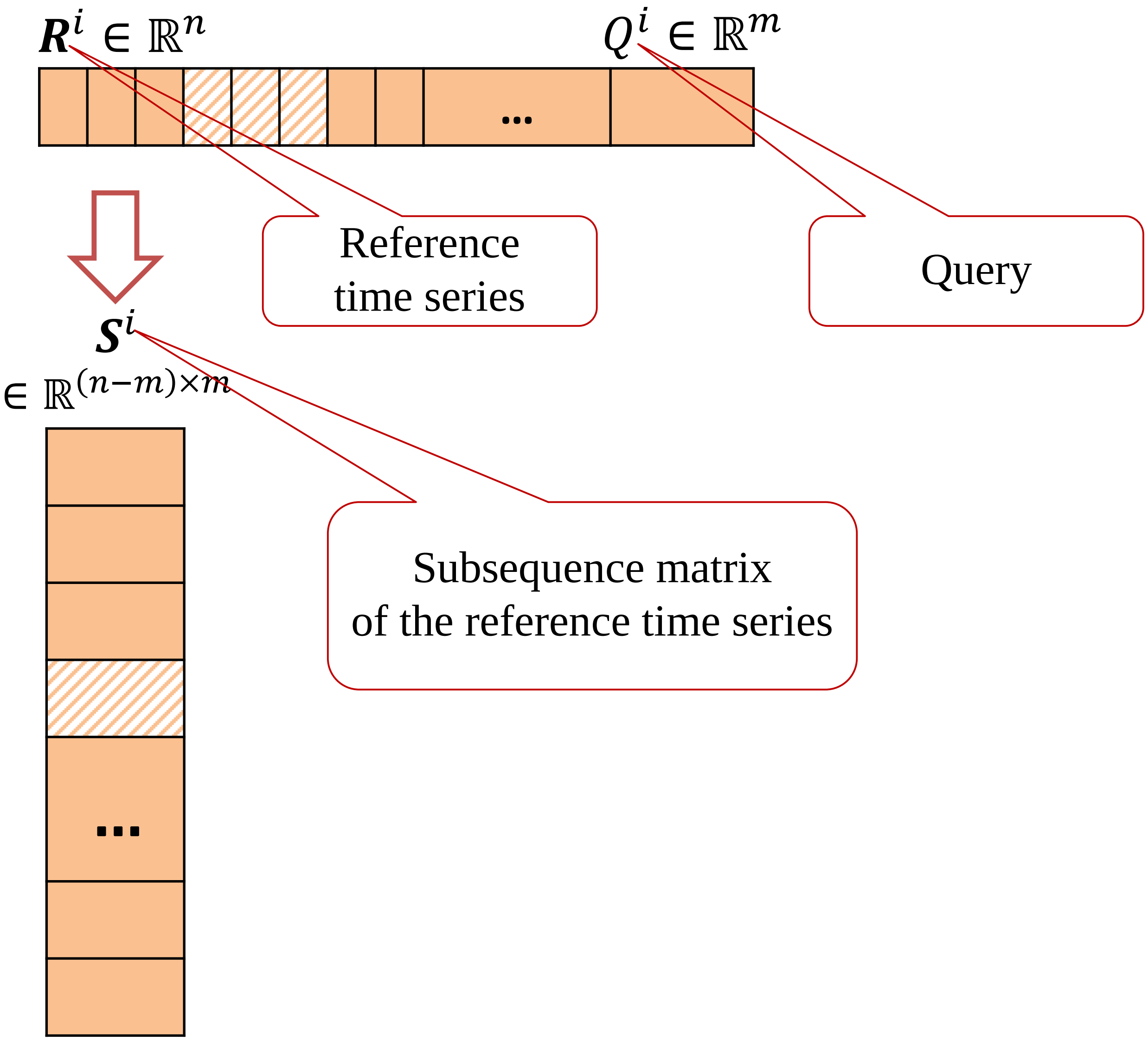
for $j := 1$ **to** $n - m + 1$ **do**

for $i := 1$ **to** d **do**

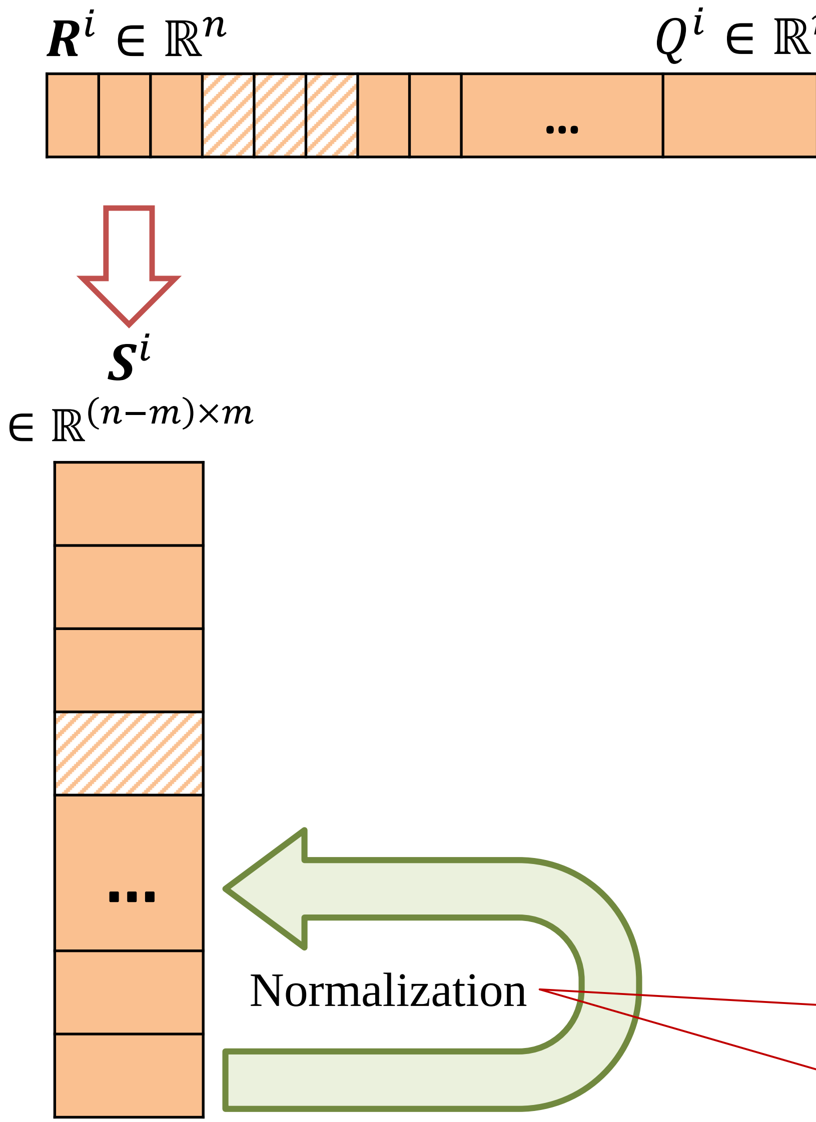
$Score(j) := Score(j) + \frac{weight(j,i)}{DTW^i(j)+\varepsilon}$

3. Selection of TOP-k intervals, reconstruction

ParaDI: Pattern search



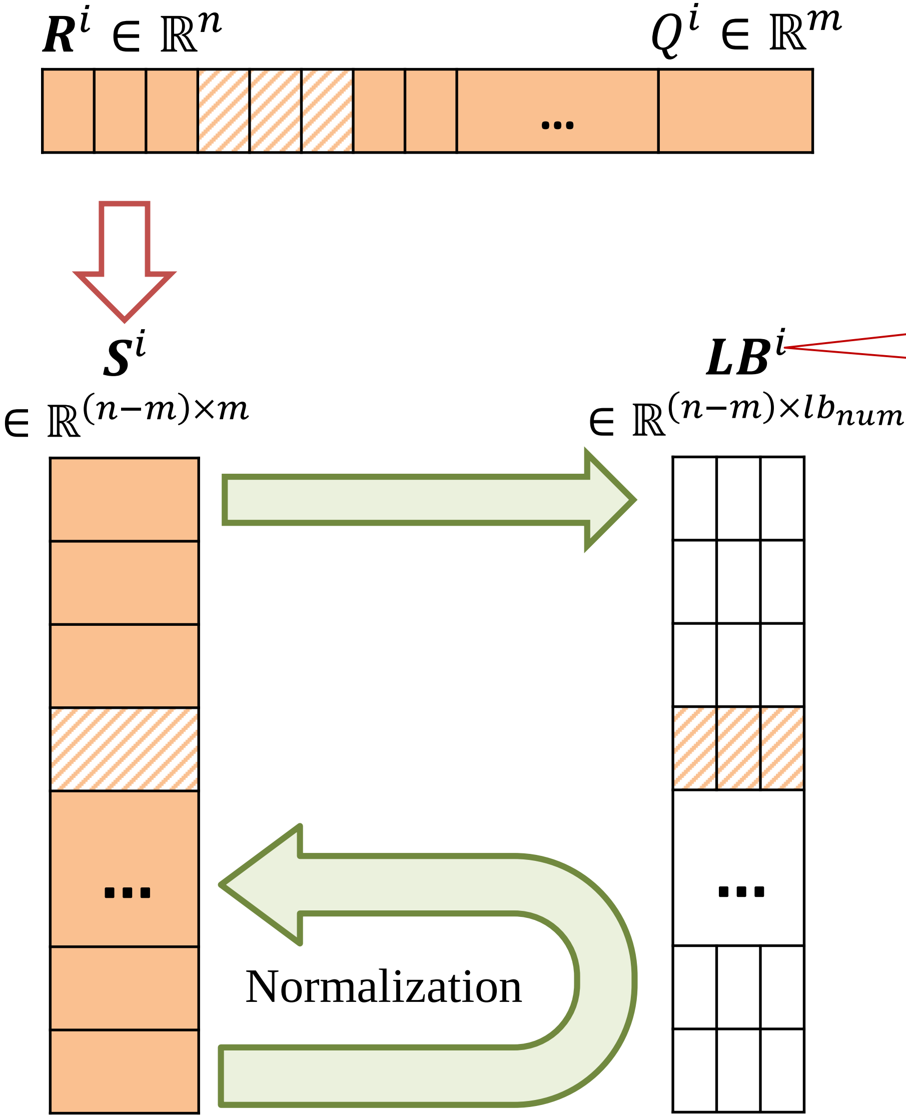
ParaDI: Normalization



$$R[1{:}m] \Rightarrow \hat{R}\,[1{:}m], \quad \hat{r}_i = \frac{r_i - \mu}{\sigma},$$

$$\mu = \frac{1}{m}\sum_{i=1}^{m} r_i, \quad \sigma^2 = \frac{1}{m}\sum_{i=1}^{m} r_i^2 - \mu^2$$

ParaDI: Lower bounds

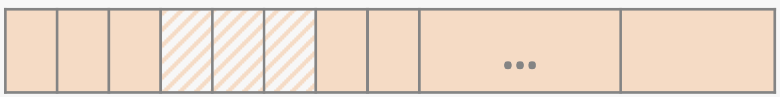


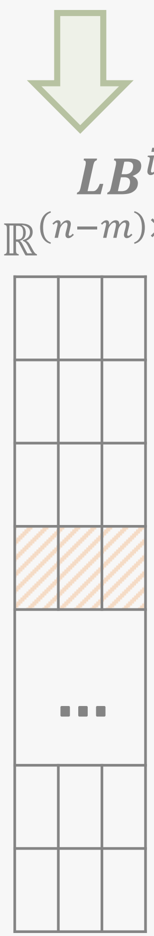
Matrix
of lower bounds

Ahead calculation of all LBs
for all subsequences
is significantly **redundant**
(cf. LB cascade in serial search)
but
it is **parallel**

ParaDI: Initializing the bsf threshold

$\boldsymbol{R}^i \in \mathbb{R}^n$ $\quad Q^i \in \mathbb{R}^m$



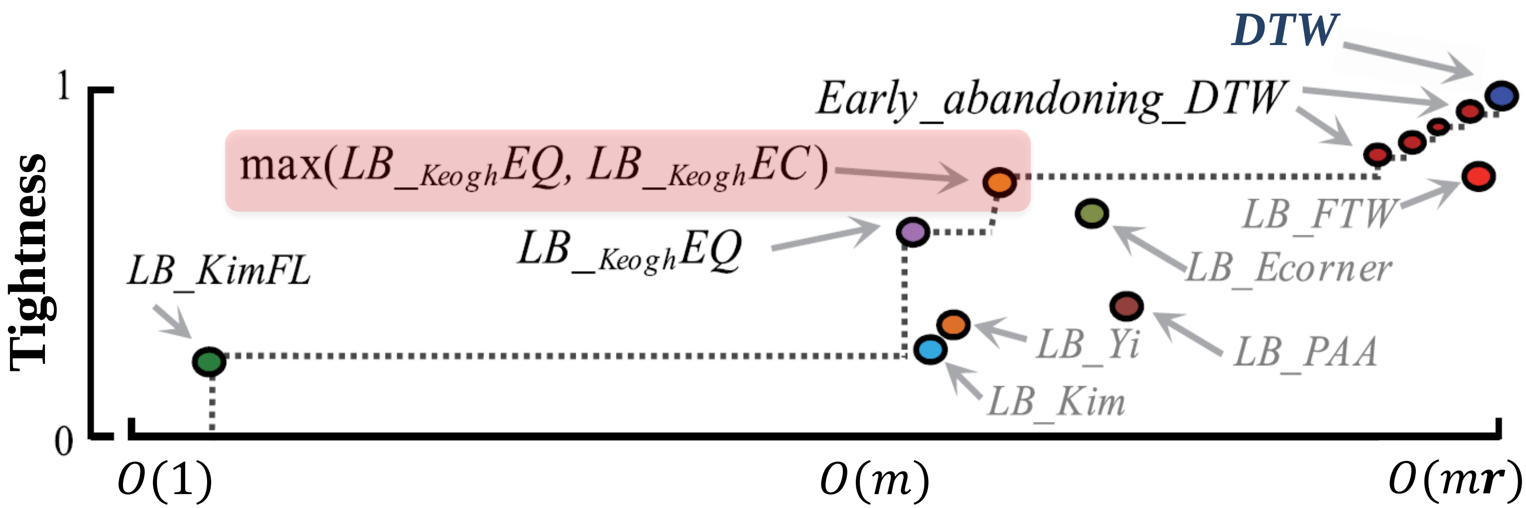


$\boldsymbol{LB}^i \in \mathbb{R}^{(n-m)\times lb_{num}}$

$$bsf_{init} \coloneqq \mathrm{DTW}(Q^i, C)$$

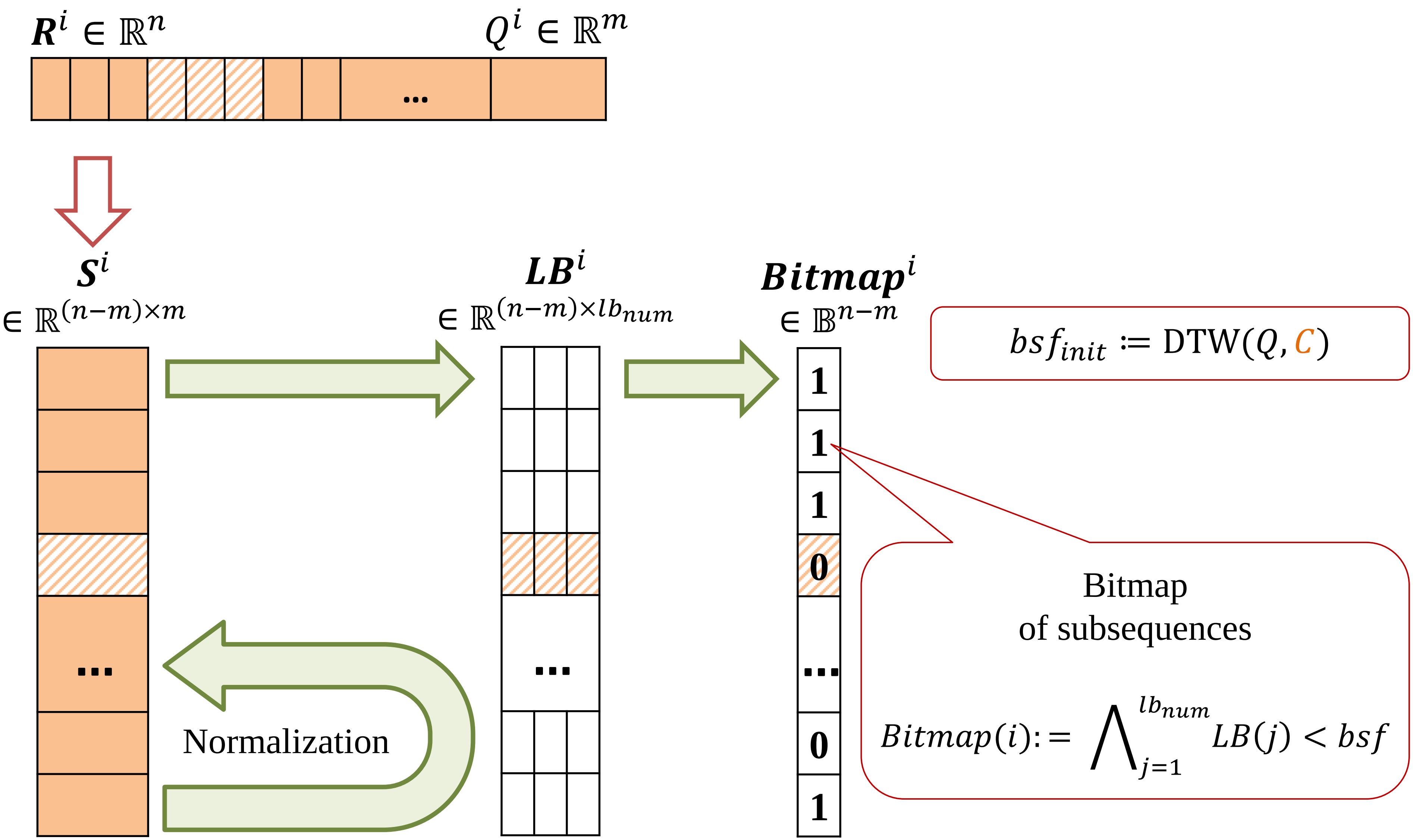
$$C = \arg \min_{1\le p\le n-m} \max_{1\le j\le lb_{num}} \boldsymbol{LB}_j^i(Q^i, \boldsymbol{R}[p{:}m])$$

$$Tightness^* = \frac{\boldsymbol{LB}(C,Q)}{\boldsymbol{DTW}(C,Q)}$$

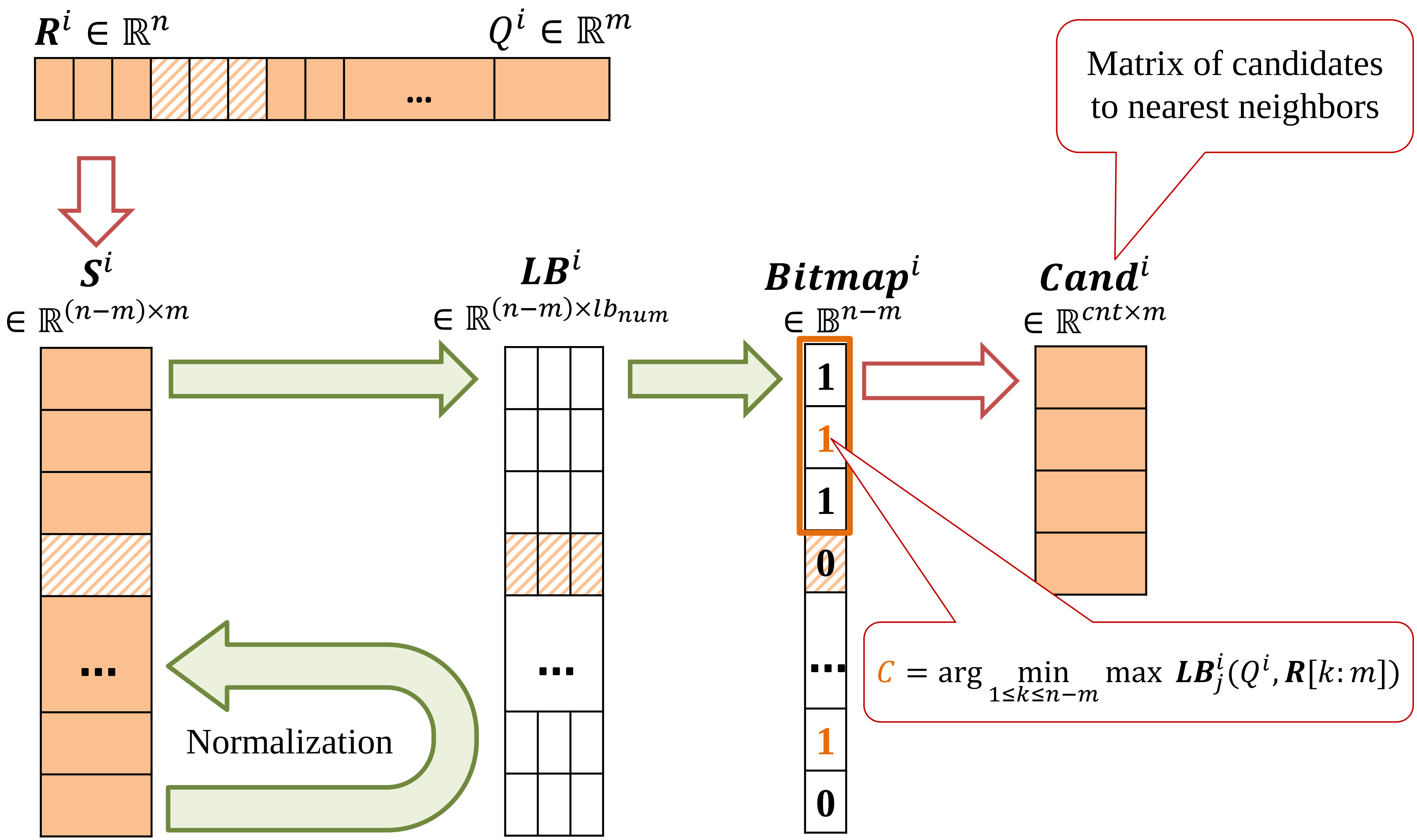


* Rakthanmanon T., *et al.* Addressing big data time series: Mining trillions of time series subsequences under Dynamic Time Warping. ACM Trans. Knowl. Discov. Data. 2013. Vol. 7, no. 3. 10:110:31. DOI: 10.1145/2500489.

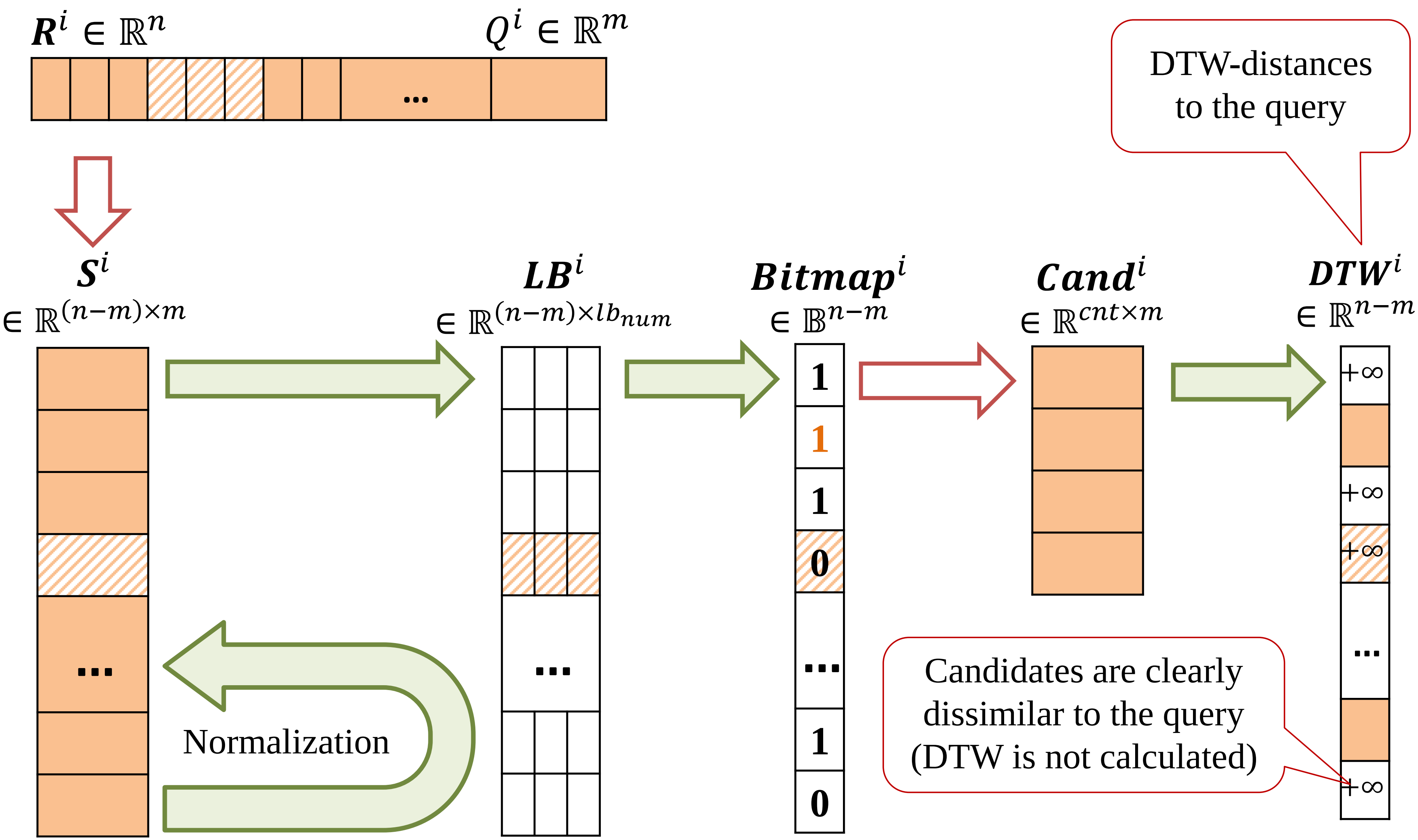
ParaDI: Lower bounding



ParaDI: Candidate matrix



ParaDI: DTW calculation



ParaDI: vectorization in DTW calculation

```
double DTW(a: array [1..m], b: array [1..m], r: int) {
   cost := array [1..m]
   cost_prev := array [1..m]

   for i := 1 to m
      cost[i] = infinity
      cost_prev[i] = infinity

   cost_prev[1] = dist(a[1], b[1])

   for j := max(2, i-r) to min(m, i+r)
      cost_prev[j] := cost_prev[j-1] + dist(a[1], b[j])

   for i := 2 to m
      for j := max(1, i-r) to min(m, i+r)
         c := d(a[i], b[j])
         cost[j] := c + min(cost[j-1], cost_prev[j-1], cost_prev[j])
      swap(cost, cost_prev)

   return cost_prev[m]
}
```

Cannot be vectorized by a compiler, ☹
but can be rewritten to be vectorized ☺

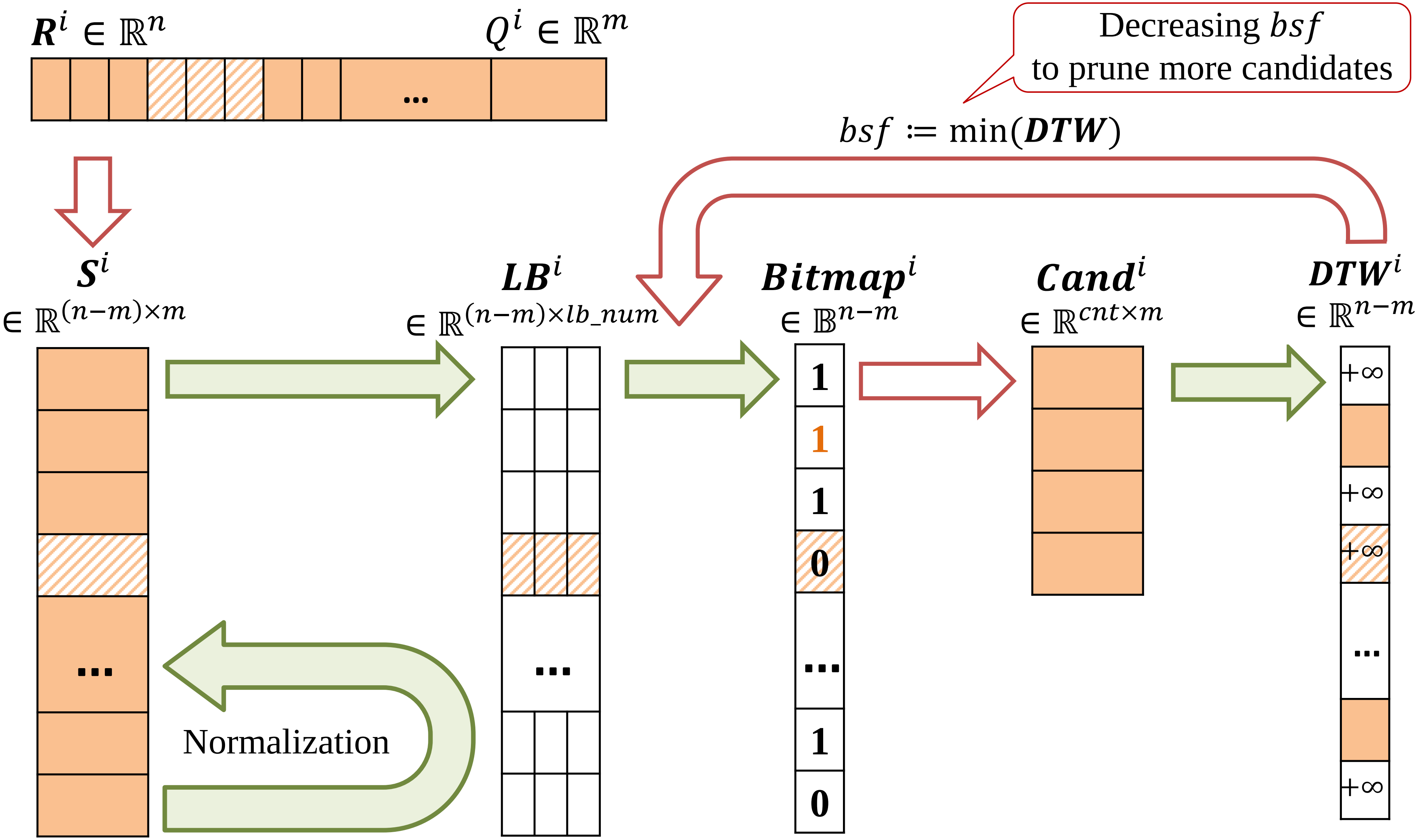
ParaDI: vectorization in DTW calculation

```
double DTW(a: array [1..m], b: array [1..m], r: int) {
    cost := array [1..m]
    cost_prev := array [1..m]
    for i := 1 to m
        cost[i] = infinity
        cost_prev[i] = infinity
    cost_prev[1] = dist(a[1], b[1])
    for j := max(2, i-r) to min(m, i+r)
        cost_prev[j] := cost_prev[j-1] + dist(a[1], b[j])
    for i := 2 to m
        for j := max(1, i-r) to min(m, i+r)
            cost[j] = min(cost_prev[j-1], cost_prev[j])
        for j := max(1, i-r) to min(m, i+r)
            c := dist(a[i], b[j])
            cost[j] := c + min(cost[j-1], cost[j])
        swap(cost, cost_prev)
    return cost_prev[m]
}
```

Vectorized by the compiler ☺

Not vectorized by the compiler ☹

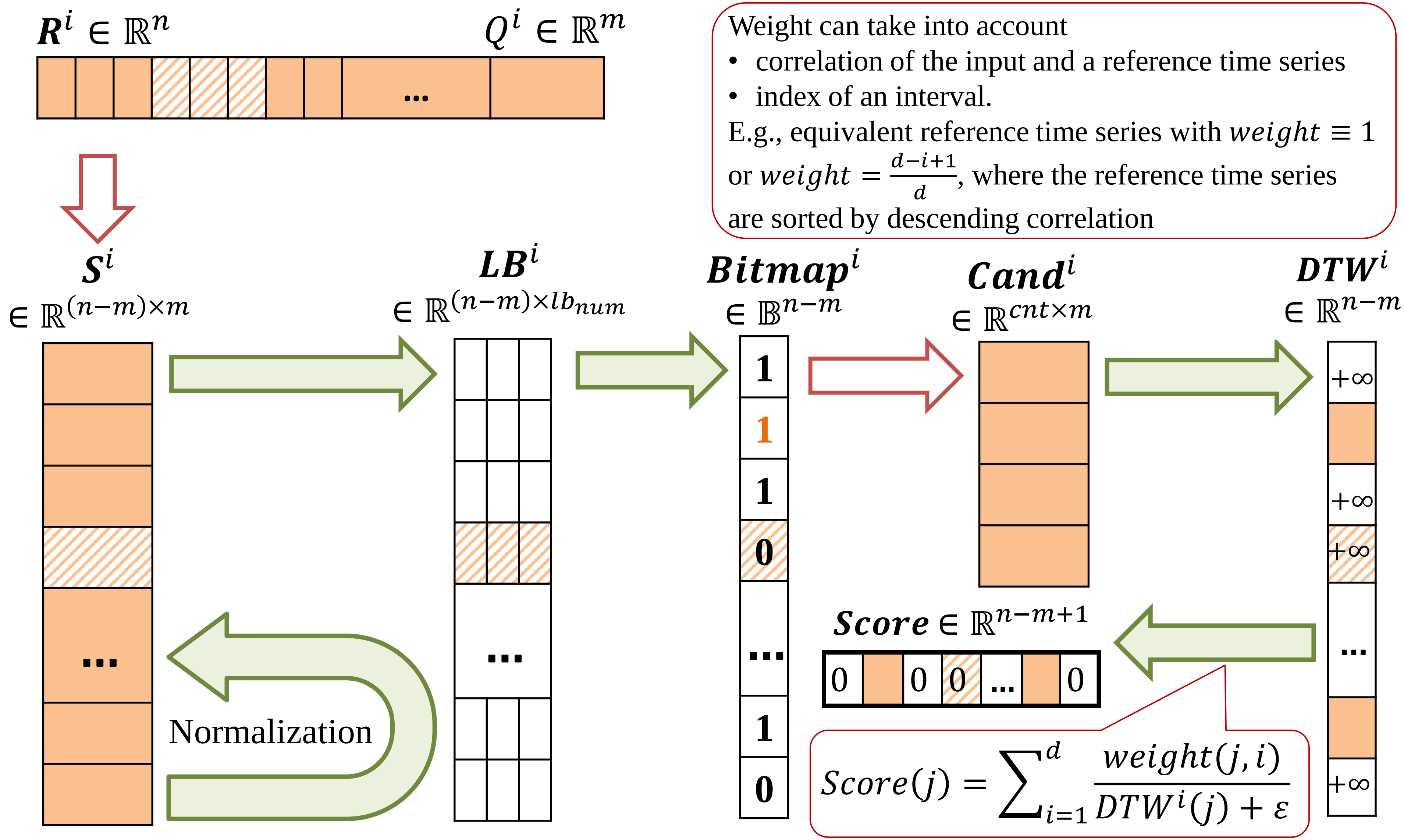
ParaDI: Improving the bsf threshold



ParaDI: Scoring

Weight can take into account
- correlation of the input and a reference time series
- index of an interval.

E.g., equivalent reference time series with $weight \equiv 1$ or $weight = \frac{d-i+1}{d}$, where the reference time series are sorted by descending correlation



ParaDI: Experiments

- **Hardware** (SUSU SSL): Intel Xeon E5-2687W v2 (8 cores @3.40 GHz)
- **Data**

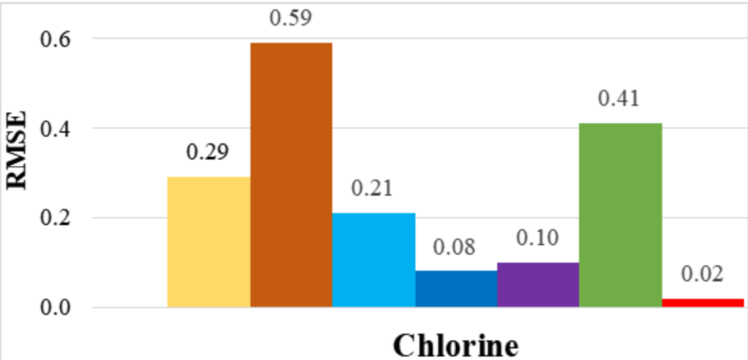
Dataset	# t.s., $d+1$	Length, $n \cdot 10^3$	Domain
BAFU	10	50	Water discharge in Swiss rivers
Chlorine	50	1	Simulation of the chlorine concentration in a drinking water system
Climate	10	5	Weather in locations of North America
MADRID	10	25	Road traffic (AVR statistics) in Madrid
MAREL	10	50	Characteristics of sea water in the English Channel

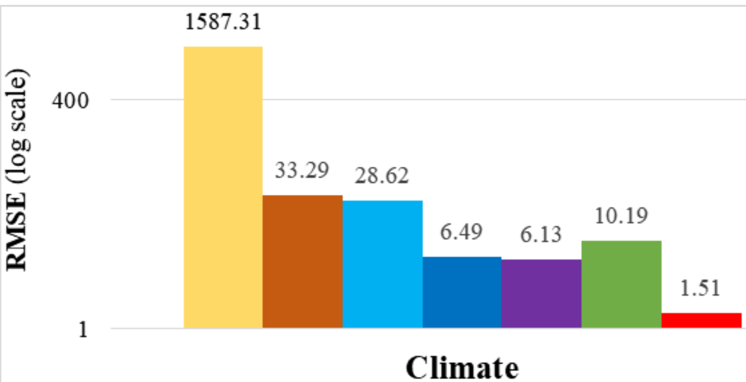
- **Rivals**: ORBITS[1], OGD-Impute[2], SPIRIT[3], SAGE[4], TKCM[5]
- **Setup** under ORBITS[1] framework:
  - Scenario: imputation of last 10% points; cases for max and min number of ref. time series
  - Rivals: best recommended parameters
  - ParaDI: $m = 50$, $k = 3$, $r = 0.25m$

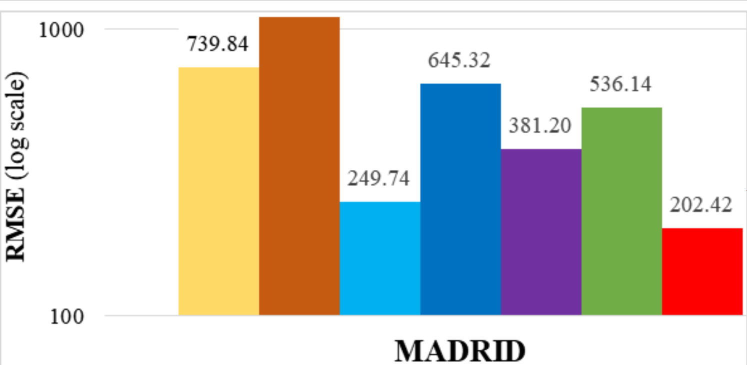
[1] Khayati M., *et al.* ORBITS: Online Recovery of Missing Values in Multiple Time Series Streams. Proc. VLDB Endow. 2020. Vol. 14, no. 3. P. 294-306.
[2] Anava O., *et al.* Online Time Series Prediction with Missing Data. Proc. ICML 2015. P. 2191-2199.
[3] Papadimitriou S., *et al.* Streaming Pattern Discovery in Multiple Time-Series. VLDB 2005. P. 697-708.
[4] Balzano L., *et al.* Streaming PCA and Subspace Tracking: The Missing Data Case. Proc. of IEEE. 2018. Vol. 106, no. 8. P. 1293-1310.
[5] Wellenzohn K., *et al.* Continuous Imputation of Missing Values in Streams of Pattern-Determining Time Series. EDBT 2017. P. 330-341.

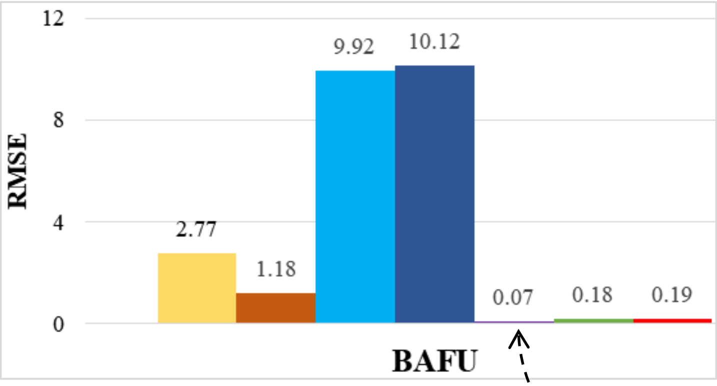
Experiments: accuracy

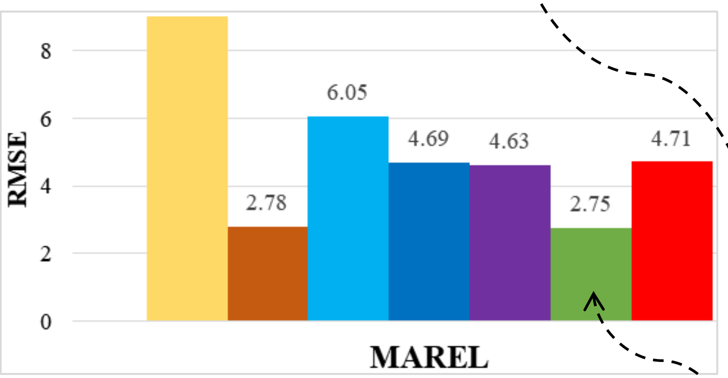
$$\text{RMSE} = \sqrt{\frac{1}{h}\sum_{i=1}^{h}(t_i - \tilde{t}_i)^2}$$











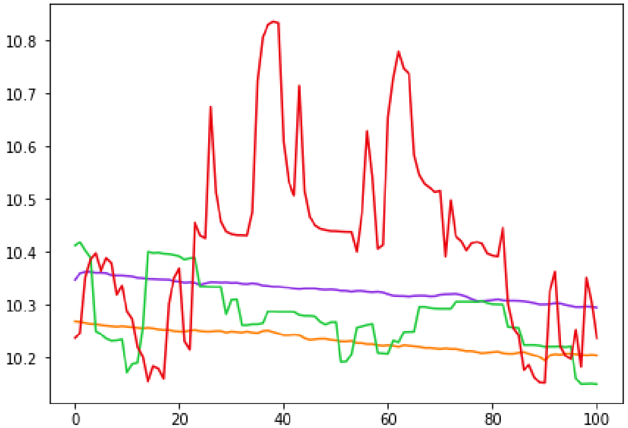
ParaDI is **ahead** of many but **not all** rivals:

SPIRIT on BAFU, TKCM on MAREL

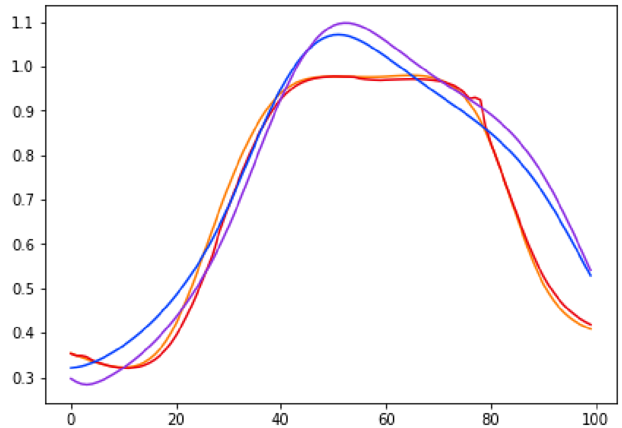
SAGE | OGD-Impute | ORBITS (k=2) | ORBITS (k=3)

SPIRIT | TKCM | **ParaDI**

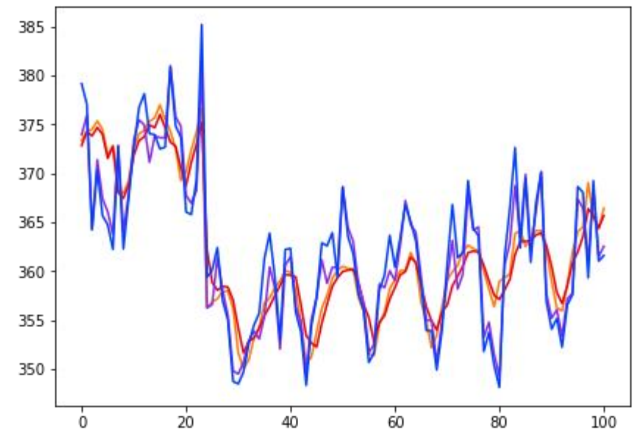
Experiments: imputation of the first 100 points*



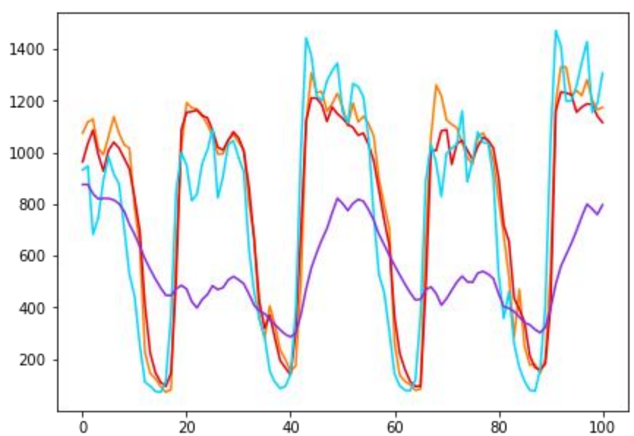
BAFU



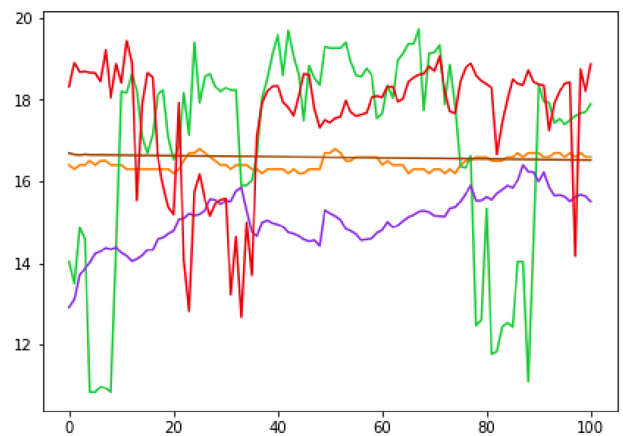
Chlorine



Climate



MADRID



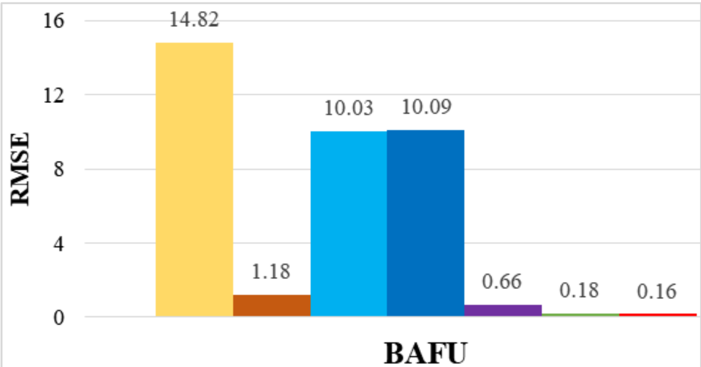
MAREL

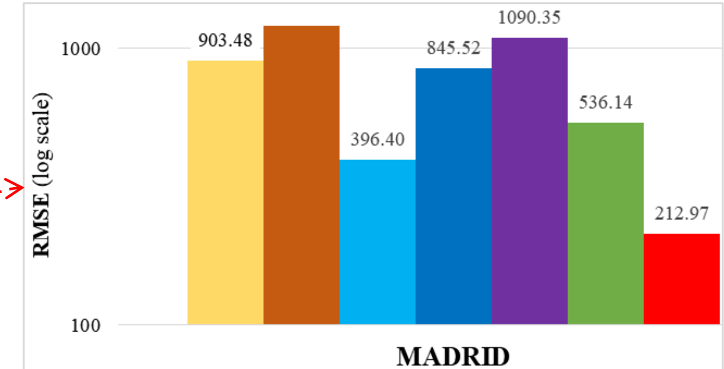
- SAGE
- OGD-Impute
- SPIRIT
- TKCM
- ORBITS (k=2)
- ORBITS (k=3)
- **ParaDI**
- **real data**

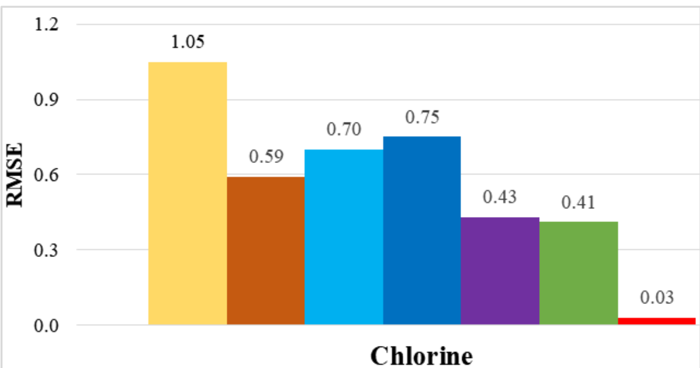
* We show the real data and the results of top-3 algorithms w.r.t. accuracy

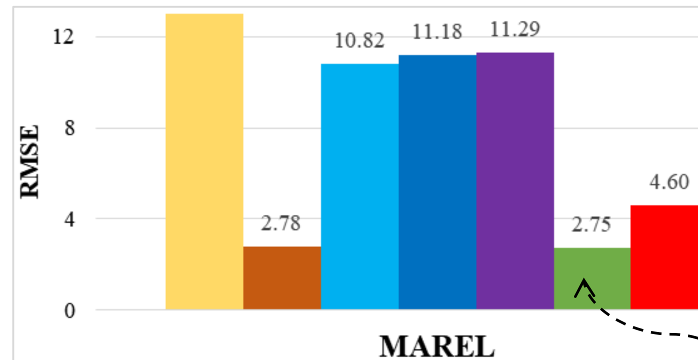
Experiments: accuracy (min # ref. t.s.)

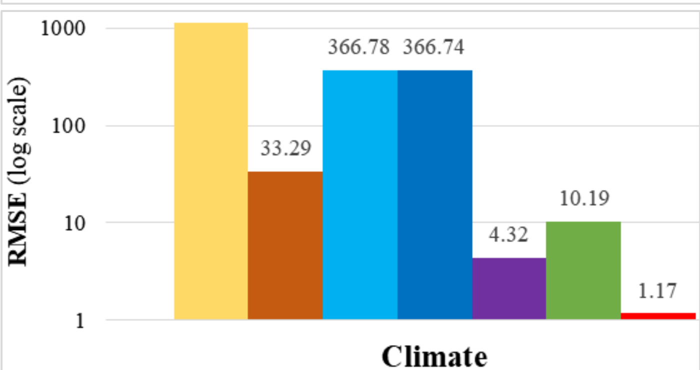
$$\text{RMSE} = \sqrt{\frac{1}{h}\sum_{i=1}^{h}(t_i - \tilde{t}_i)^2}$$







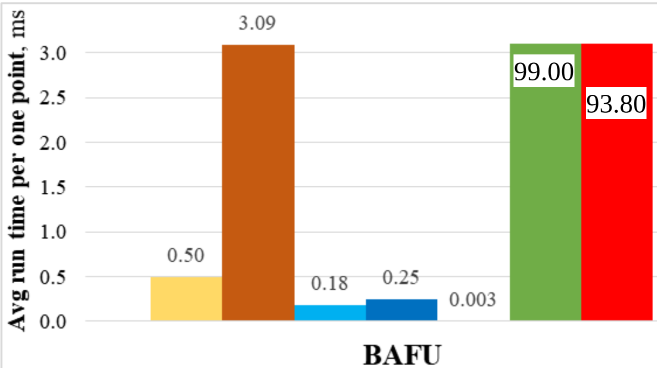


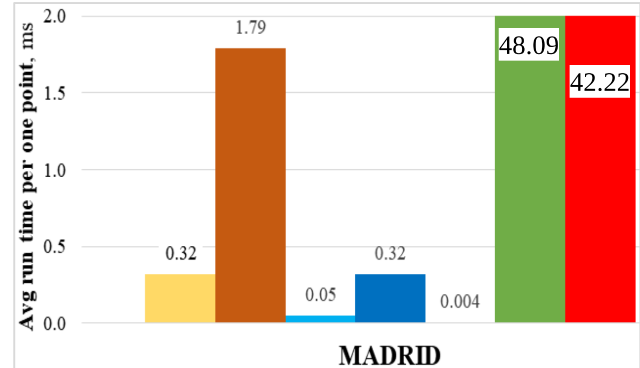


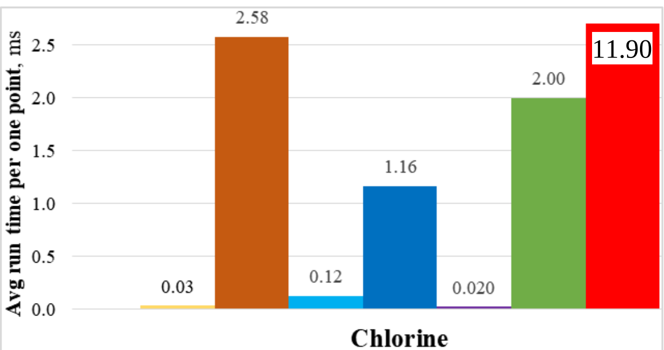
ParaDI is **ahead** of all but **one** rival: TKCM on MAREL

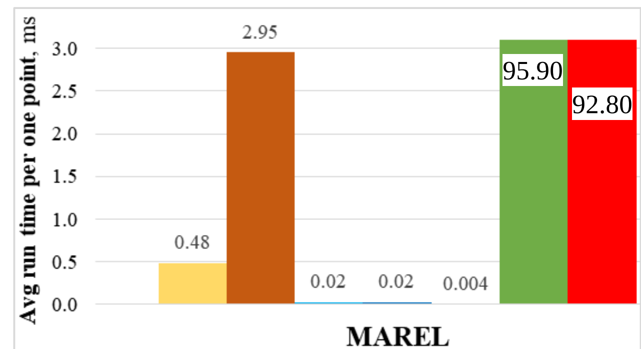
SAGE | OGD-Impute | ORBITS (k=2) | ORBITS (k=3)
SPIRIT | TKCM | **ParaDI**

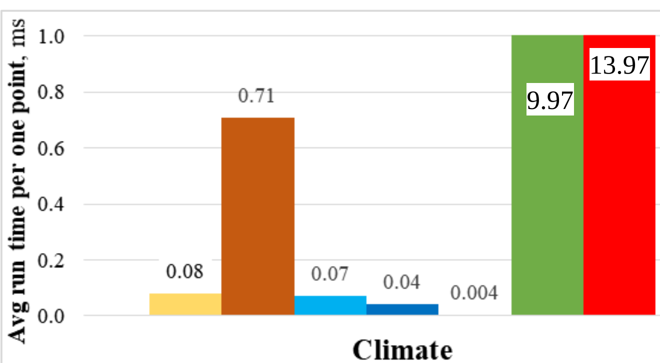
Experiments: performance







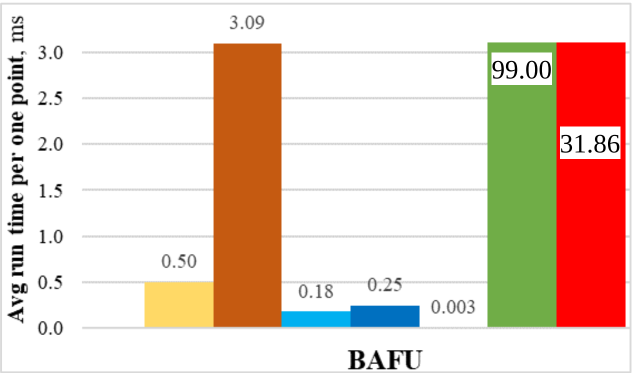


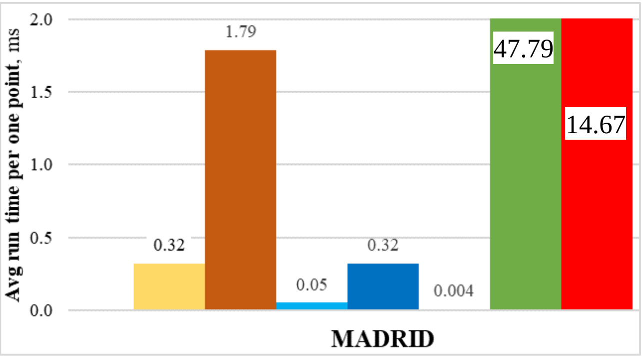


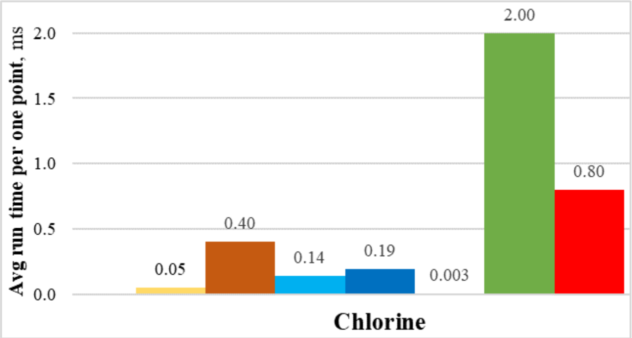
ParaDI is **inferior to all but one rival** (TKCM)

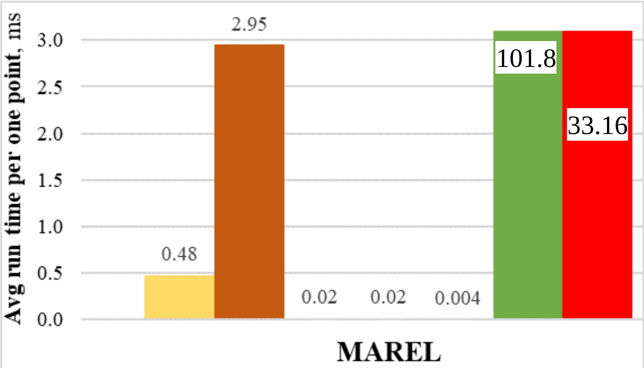
SAGE OGD-Impute ORBITS (k=2) ORBITS (k=3)
SPIRIT TKCM **ParaDI**

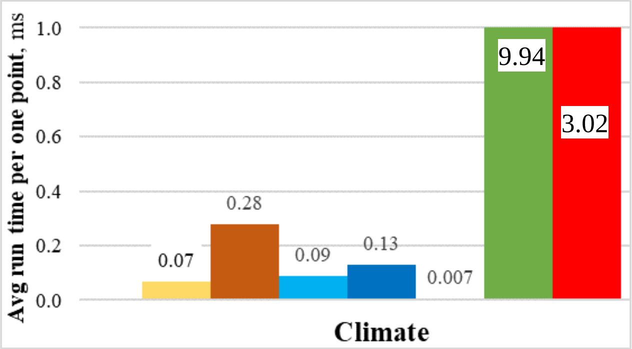
Experiments: performance (min # ref. t.s.)







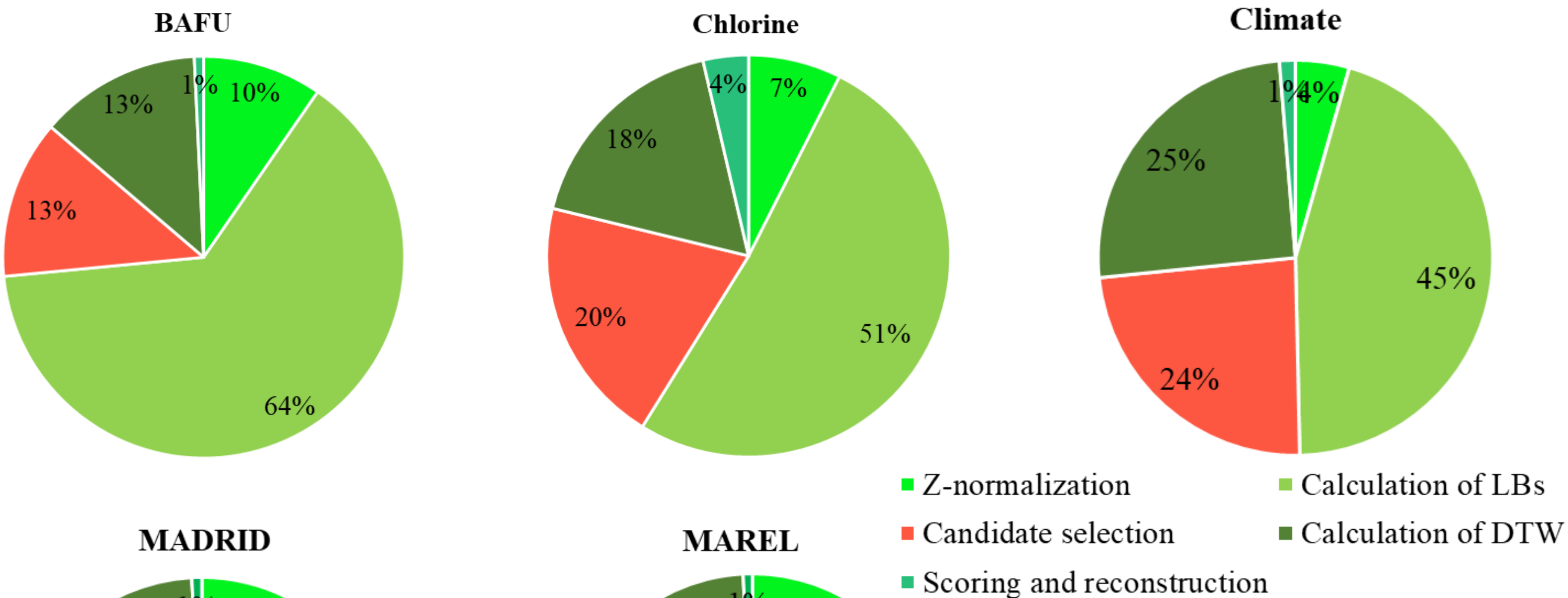


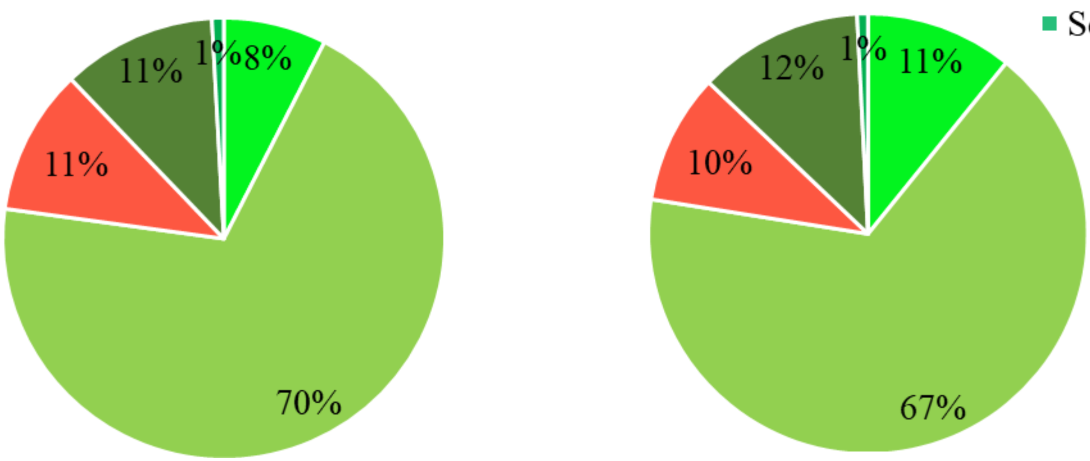


ParaDI is **inferior to all but one rival** (TKCM), SPIRIT is the fastest

SAGE | OGD-Impute | ORBITS (k=2) | ORBITS (k=3)
SPIRIT | TKCM | **ParaDI**

Experiments: runtime spent by phases

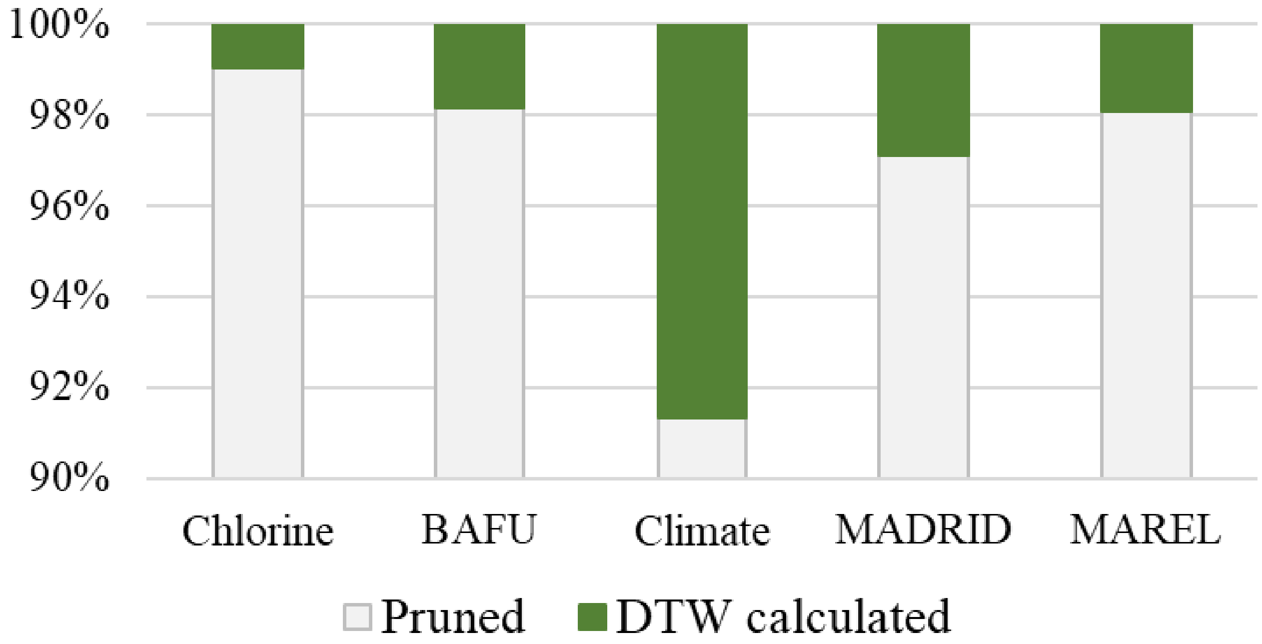




Top-3
time consuming phases:

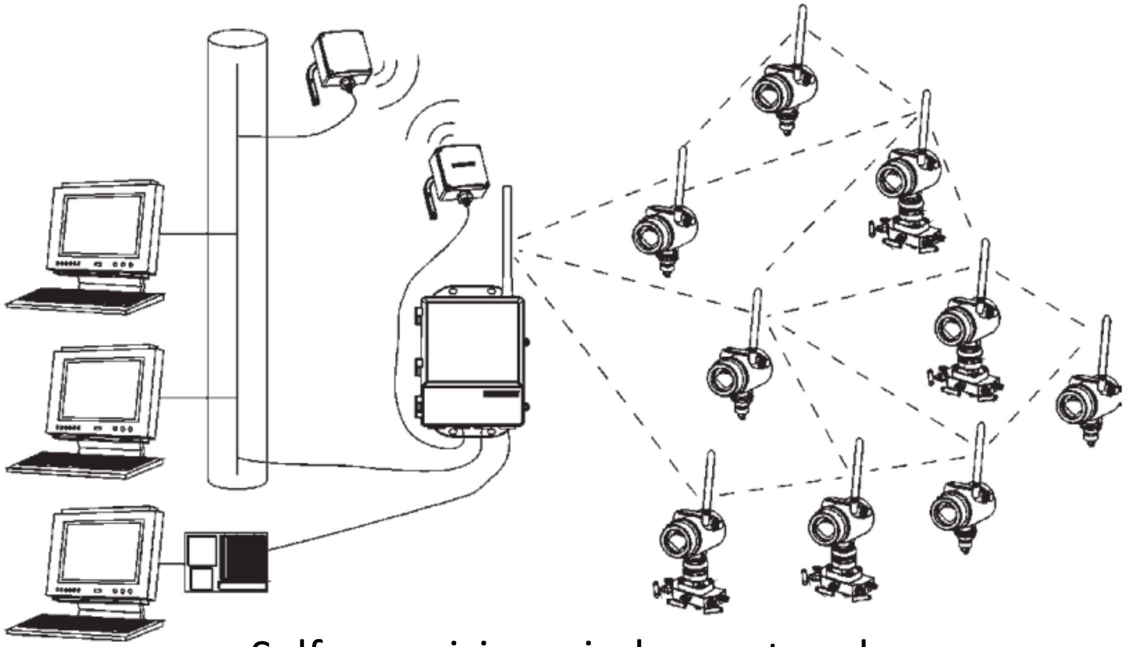
1. **Calculation of LBs**
2. **Calculation of DTW**
3. **Candidate selection**

Experiments: efficiency of lower bounding



From 91% to 99% calculations of DTW are pruned

ParaDI: as for real-time mode…



Self-organizing wireless network

Wireless Gateway 1420*		**ParaDI**, **ms**	
# sensors	min data update time, **s**	worst	best
100	8	93.8	0.8
50	4		
12	1		

ParaDI fits real-time mode

* Emerson temperature sensors catalogue 2021. URL: https://www.c-o-k.ru/library/catalogs/emerson/110477.pdf

ParaDI: *pro et contra*

- Pros
  - ahead of many (but not all) analogs w.r.t. accuracy
  - fits for real-time mode
  - needs a minimum number of reference time series
- Cons
  - memory overhead
  - missing values in a reference time series
- Scope
  - stationary time series (the mean and variance are relatively constant)

Conclusions and further research

- ParaDI is novel parallel algorithm for imputation missing values of time series
  - Accuracy: ahead of many but not all analogs
  - Speed: inferior to all but one analog, but still fits for real-time
  - Scope: stationary time series
- Future works
  - Avoiding redundant calculations when sliding in z-norm, LBs
  - Calibration phase: (semi-)automatic choice of n, m, r, k
  - GPU version: calculation of LBs
  - Extensive experiments: impact of n, m, r, k
  - Case of missing values in a reference time series

Thank you for paying attention! Questions?

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