

High-performance subsequence anomaly discovery in long time series

明智是了解事件的人

Wise is the person who understands events.

Chinese proverb

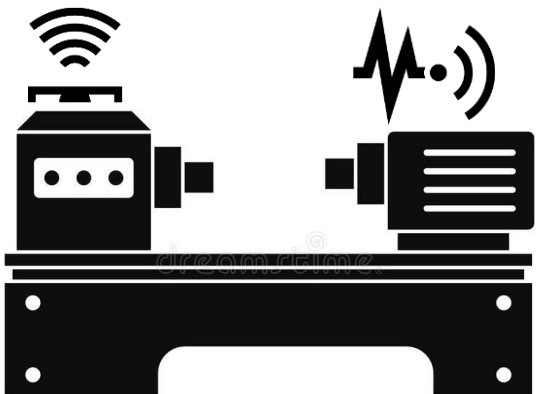


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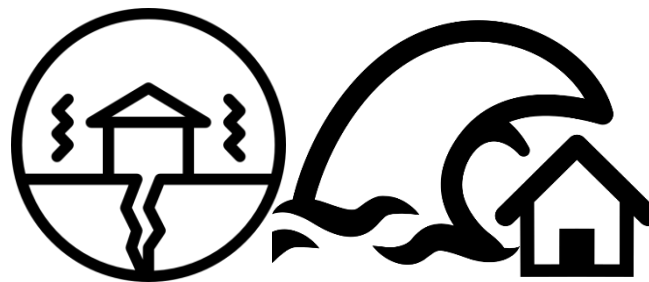
Time series are ubiquitous



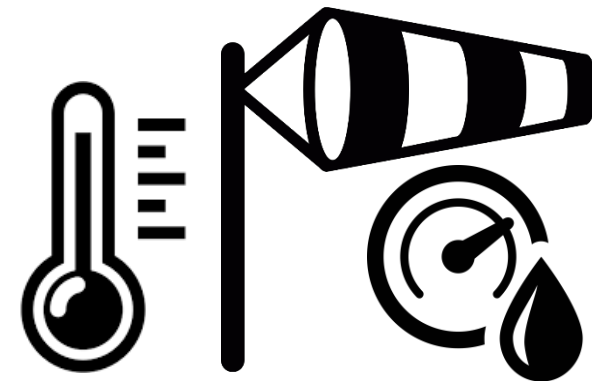
**Smart manufacturing,
Predictive maintenance**



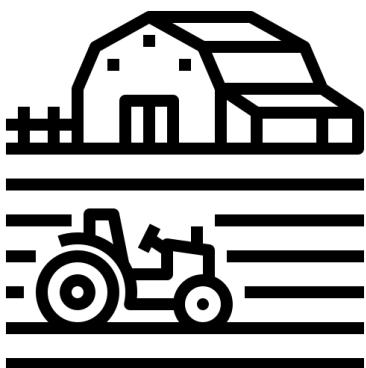
**Internet
of Things**



**Prediction
of natural disasters**



**Weather forecasting,
Climate modelling**



**Agriculture
and farming**



**Personal
healthcare**



**Chemo-
and bioinformatics**



**Fighting
crime**

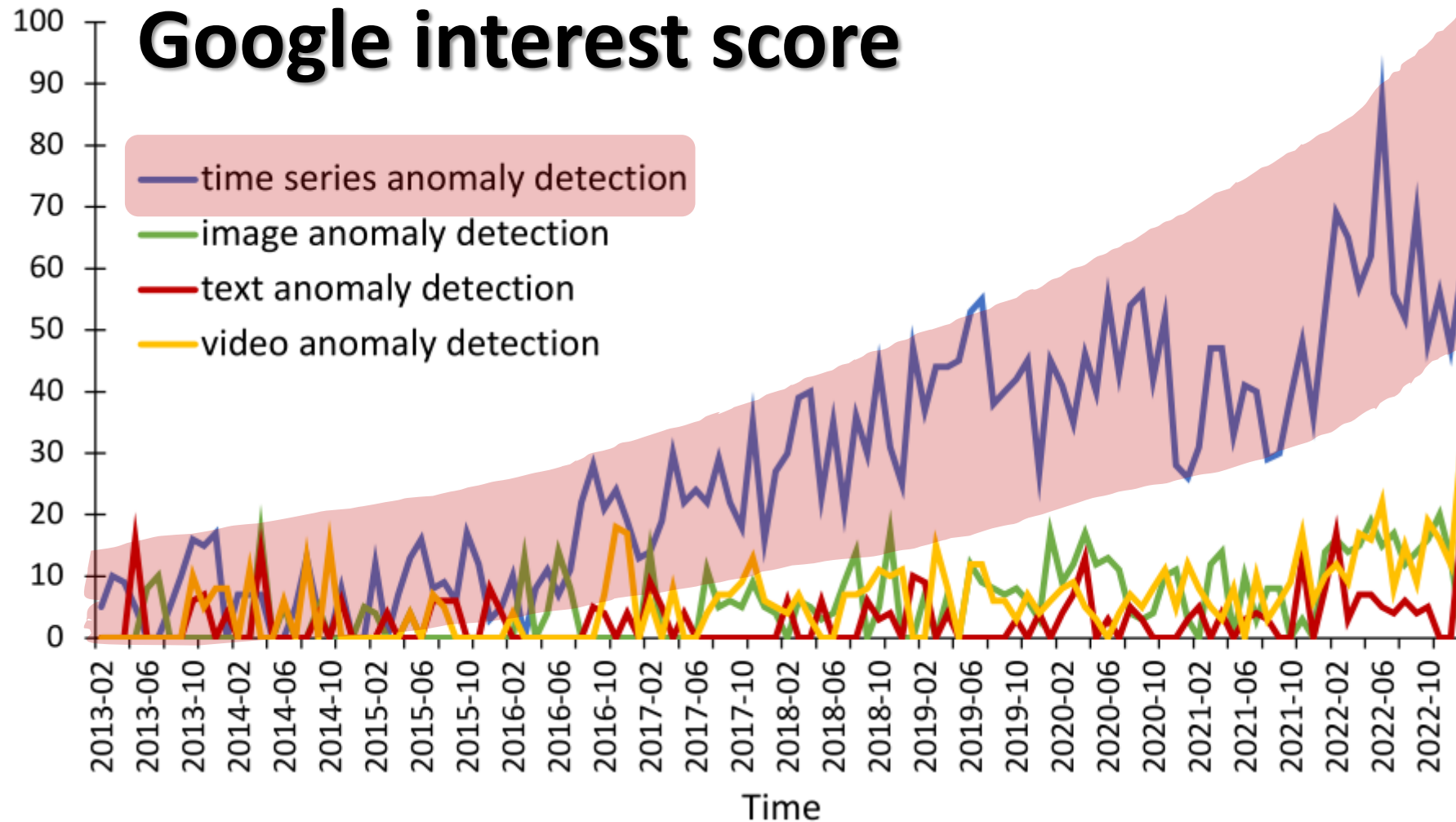


**Business
and economics**



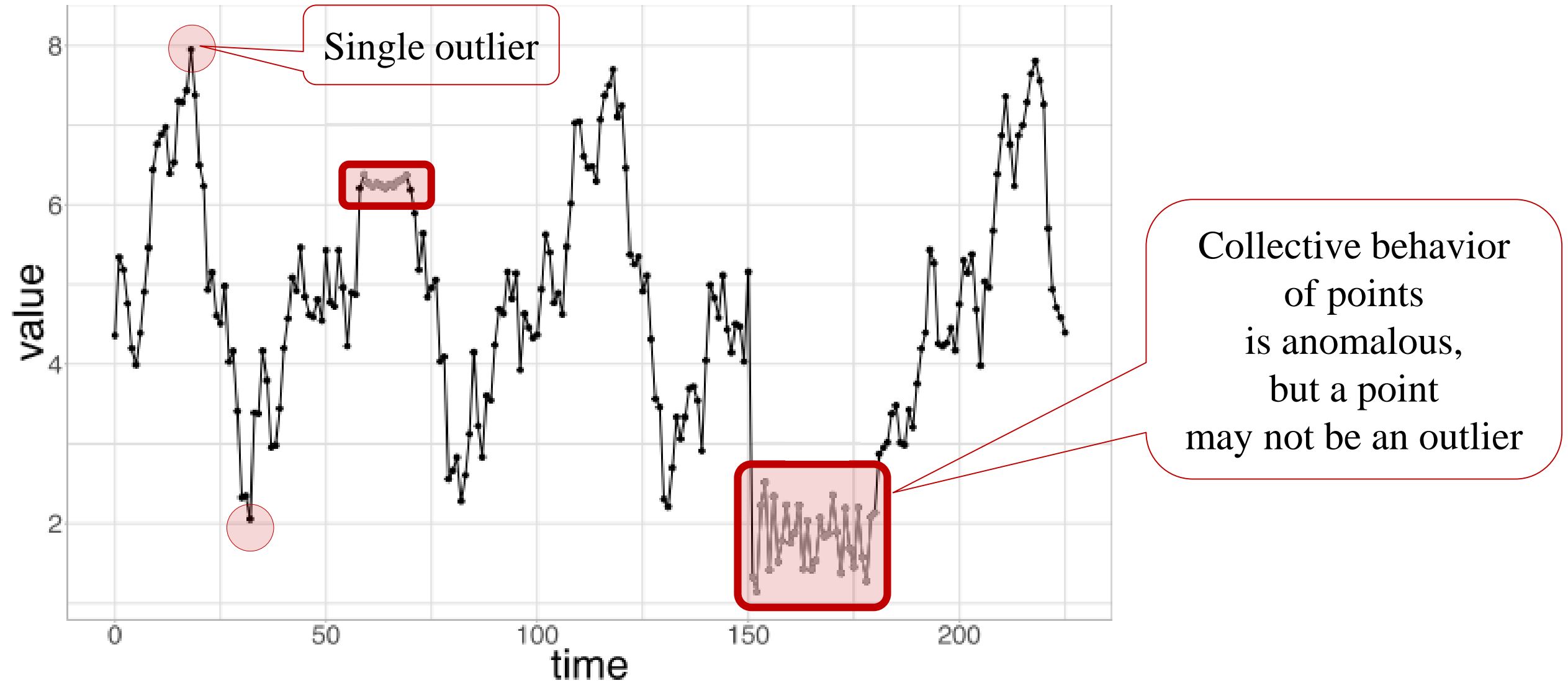
**Electronic
education**

In time series, people are most interested in anomalies



* Boniol P. *et al.* New trends in time-series anomaly detection. EDBT'2023. pp. 847-850. DOI: [10.48786/edbt.2023.80](https://doi.org/10.48786/edbt.2023.80)

Subsequence anomaly is the challenge



Anomalies are informal and domain-dependent



Homer



Marge



Bart



Selma



Patty



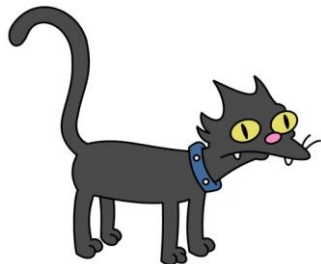
Barney



Quasimodo



Itchy



Scratchy



Santa's Little Helper



Lisa



Mona



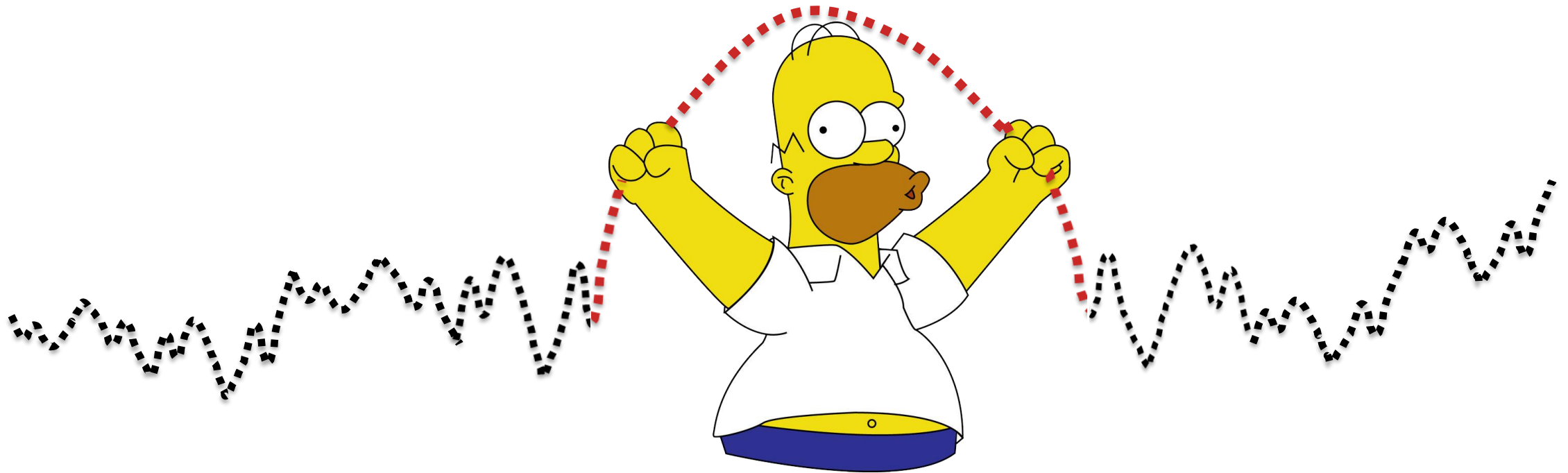
Abe



Turnip

Discord formalizes subsequence anomaly

Discord* is a subsequence of the given length whose distance to its nearest neighbor is greatest



* Keogh E. *et al.* HOT SAX: Efficiently finding the most unusual time series subsequence. ICDM 2005. pp. 226-233. DOI: [10.1109/ICDM.2005.79](https://doi.org/10.1109/ICDM.2005.79)

Discord concept

Homer



Marge



Bart



Selma



Patty



Barney



Quasimodo



Distance matrix:
the close neighbors,
the similar they are

	Homer	Marge	Bart	Selma	Patty	Barney	Quasimodo
Homer	0						
Marge		0					
Bart			0				
Selma				0			
Patty					0		
Barney						0	
Quasimodo							0

Discord concept

Homer



Marge



Bart



Selma



Patty



Barney



Quasimodo



Distance matrix
with calculated distances
to neighbors

	Homer	Marge	Bart	Selma	Patty	Barney	Quasimodo
Homer	0	5	2	4	4	6	8
Marge	5	0	2.5	3	3	6	10
Bart	2	2.5	0	4	4	6	9
Selma	4	3	4	0	0.5	5	8
Patty	4	3	4	0.5	0	5	8
Barney	6	6	6	5	5	0	7
Quasimodo	8	10	9	8	8	7	0

Discord concept

Homer



Marge



Bart



Selma



Patty



Barney



Quasimodo



	Homer	Marge	Bart	Selma	Patty	Barney	Quasimodo
Homer	0	5	2	4	4	6	8
Marge	5	0	2.5	3	3	6	10
Bart	2	2.5	0	4	4	6	9
Selma	4	3	4	0	0.5	5	8
Patty	4	3	4	0.5	0	5	8
Barney	6	6	6	5	5	0	7
Quasimodo	8	10	9	8	8	7	0

Distance matrix
with
**distances to their
nearest neighbors**
(i.e. column-wise minima)

Discord concept

Homer



Marge



Bart



Selma



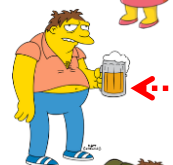
Patty



Barney



Quasimodo



Distance matrix
with the
farthest distance
to the nearest neighbor
(i.e. maximum
among
column-wise minima)

	Homer	Marge	Bart	Selma	Patty	Barney	Quasimodo
Homer	0	5	2	4	4	6	8
Marge	5	0	2.5	3	3	6	10
Bart	2	2.5	0	4	4	6	9
Selma	4	3	4	0	0.5	5	8
Patty	4	3	4	0.5	0	5	8
Barney	6	6	6	5	5	0	7
Quasimodo	8	10	9	8	8	7	0



Discord concept

Homer



Marge



Bart



Selma



Patty



Barney



Quasimodo

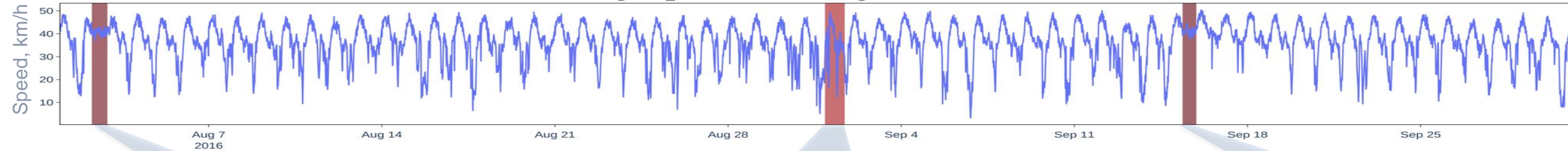


0	5	2	4	4	6	8
5	0	2.5	3	3	6	10
2	2.5	0	4	4	6	9
4	3	4	0	0.5	5	8
4	3	4	0.5	0	5	8
6	6	6	5	5	0	7
8	10	9	8	8	7	0

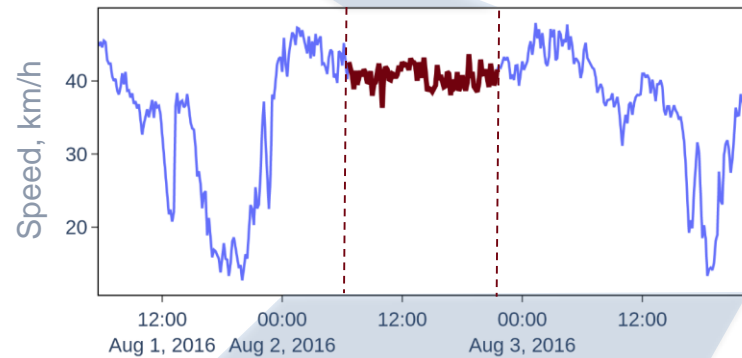
Discord is an object with the **farthest nearest neighbor** (i.e. argument of the maximum among column-wise minima)

Discords reflect anomalies in real life

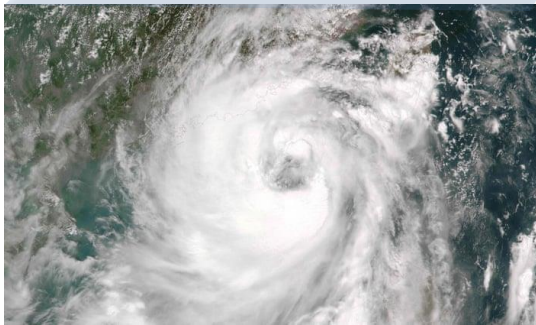
Average speed in Guangzhou, China*



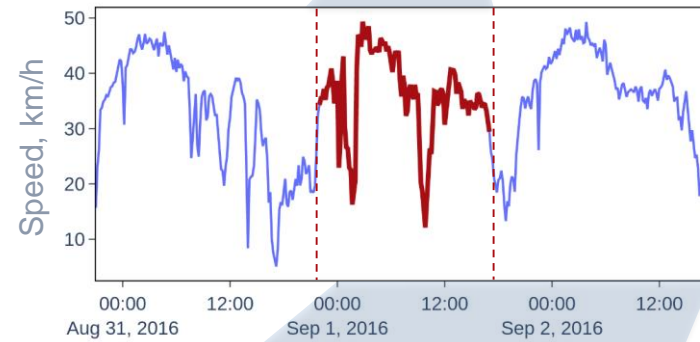
Top-2 discord



Typhoon Nida



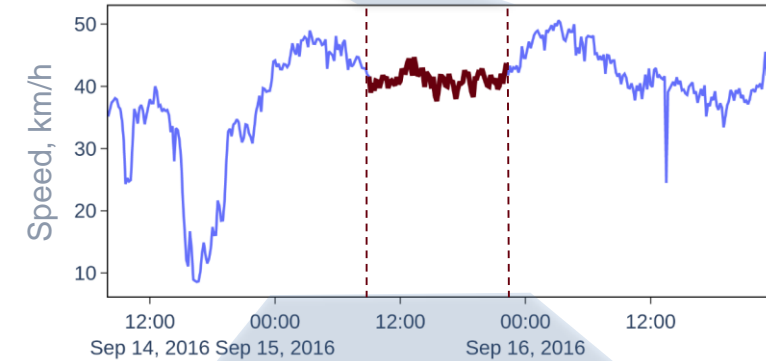
Top-3 discord



Day of Victory over Japan



Top-1 discord



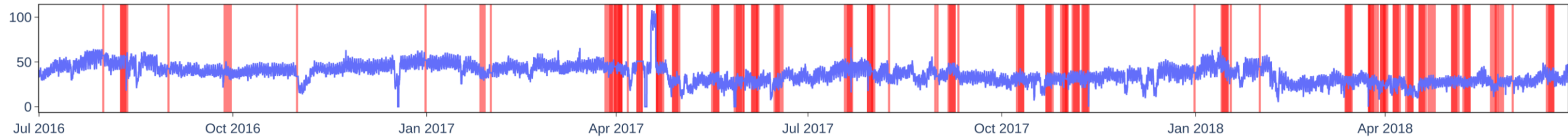
Mid-Autumn Festival



* Chen X, Chen Y, He Z. Urban traffic speed dataset of Guangzhou, China. 2018. DOI: [10.5281/zenodo.1205229](https://doi.org/10.5281/zenodo.1205229).

For long series, we need **all-length** subsequence anomalies!

Two-year power demand (Beijing Guowang Fuda Sci. & Tech. Dev. Co.)*

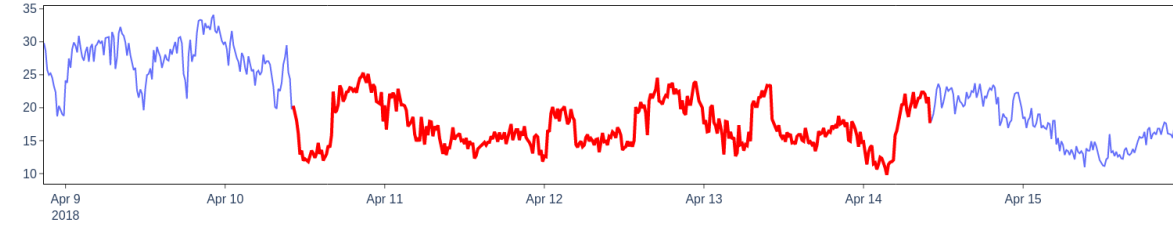
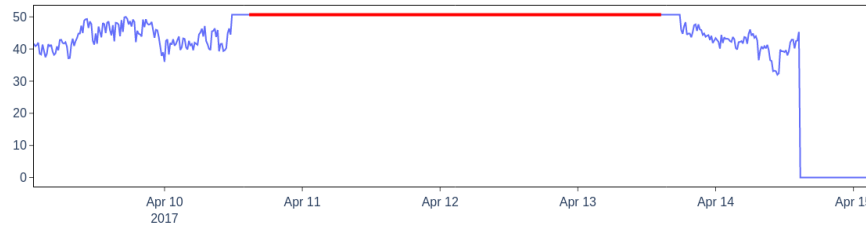
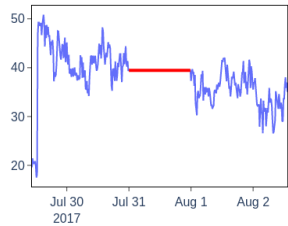


1 day

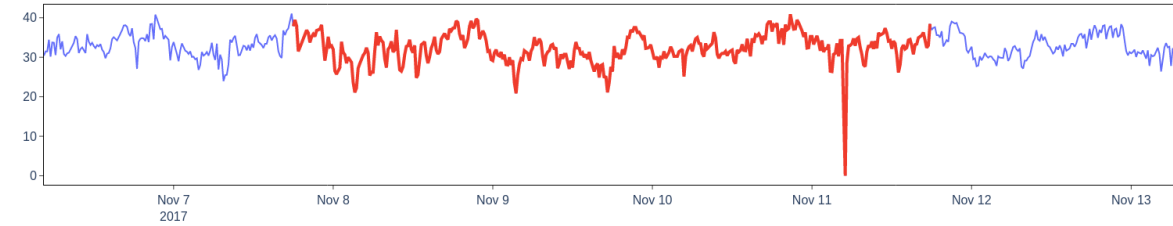
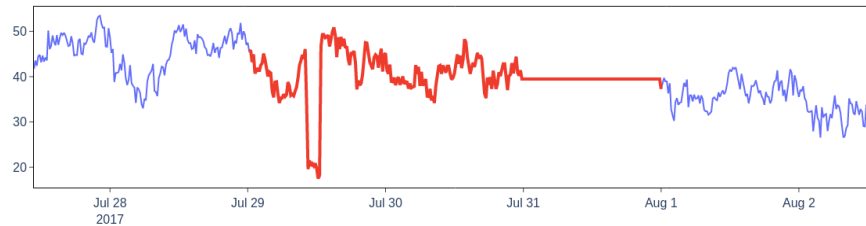
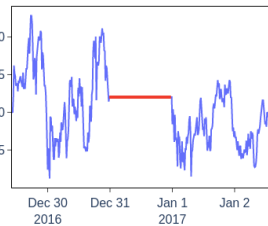
3 days

4 days

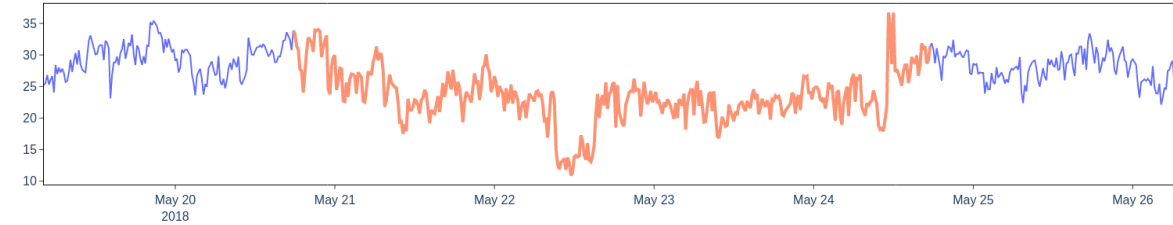
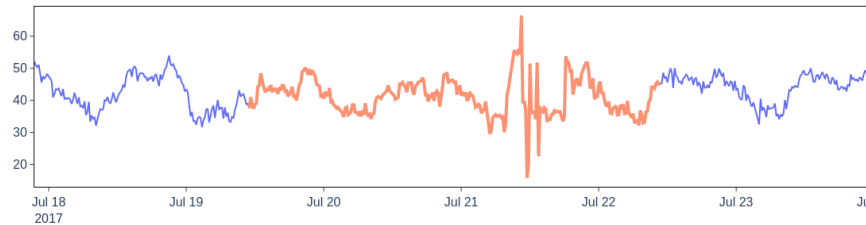
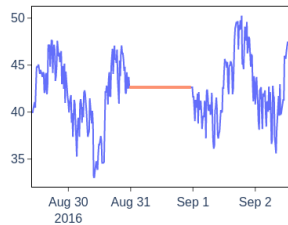
Top-1 anomaly



Top-2 anomaly



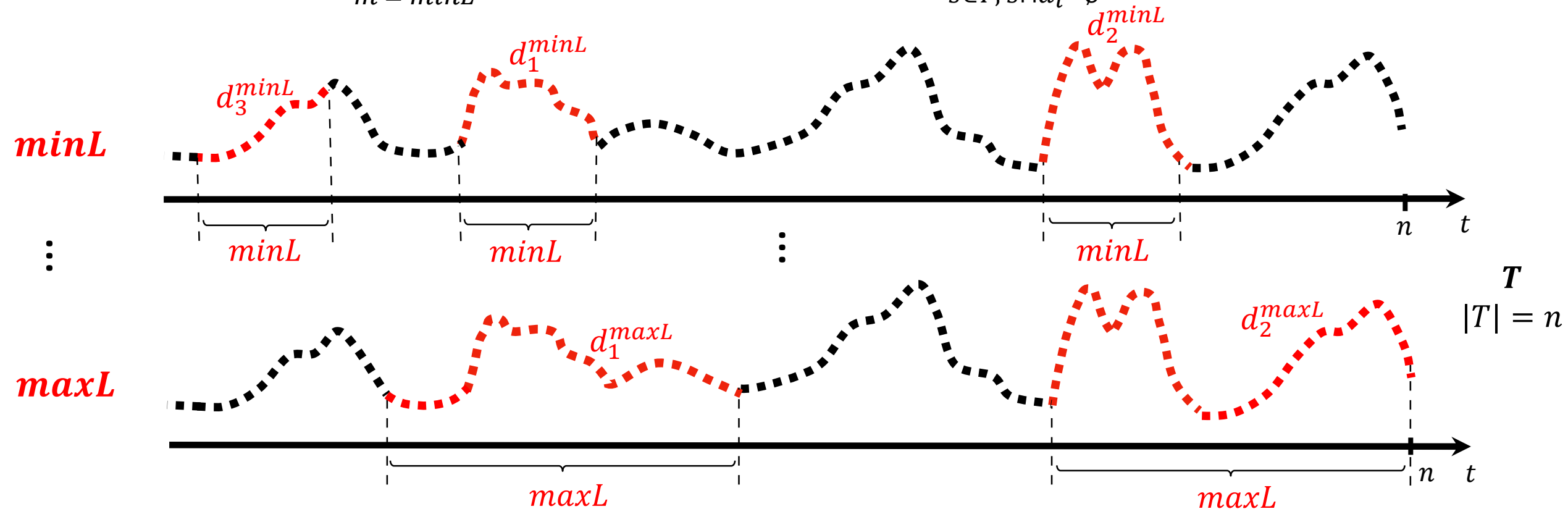
Top-3 anomaly



* Zhou H. *et al.* Informer: beyond efficient transformer for long sequence time-series forecasting. AAI 2021. pp. 11106-11115. DOI: [10.1609/aaai.v35i12.17325](https://doi.org/10.1609/aaai.v35i12.17325).

For all-length anomaly subsequence discovery, we need range discords

- **Range discord*** is a discord that is at least a **threshold** r away from its nearest neighbor
- We are given: time series T , range of discord length $minL \dots maxL$
- We are to find: $\mathcal{D} = \bigcup_{m=minL}^{maxL} D_m$, $D_m = \{d_1^m, d_2^m, \dots\}$, where $\min_{s \in T, s \cap d_i = \emptyset} \text{dist}(d_i^m, s) \geq r$



* Yankov D. et al. Disk aware discord discovery: finding unusual time series in terabyte sized datasets. ICDM 2007. pp. 381-390. DOI: [10.1109/ICDM.2007.61](https://doi.org/10.1109/ICDM.2007.61).

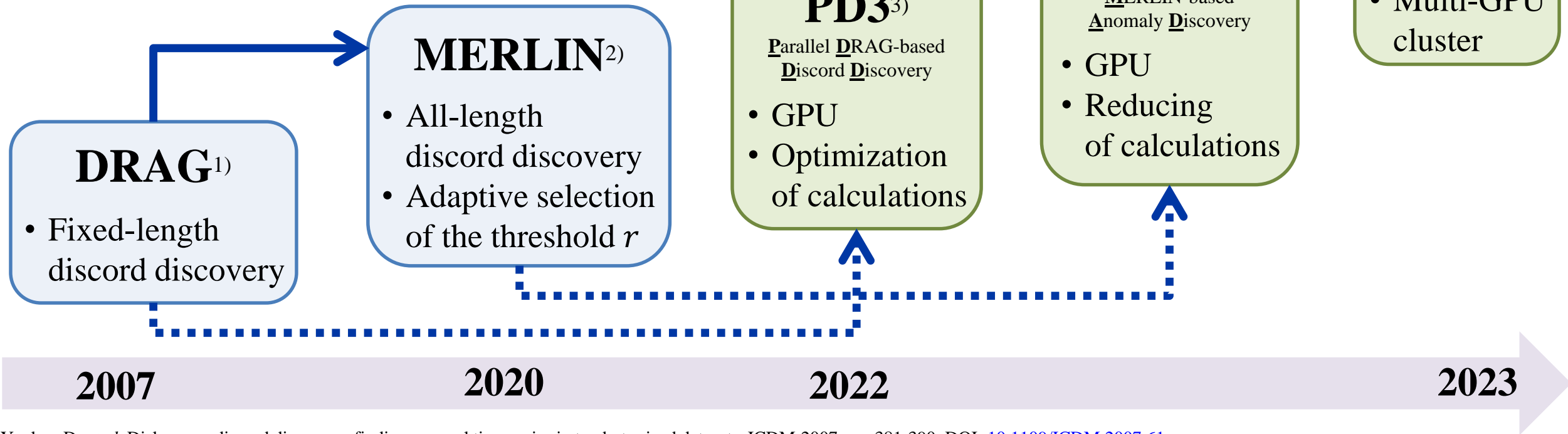
Discord discovery algorithms

Serial algorithm

Keogh *et al.*

Parallel algorithm

Zymbler and Kraeva



¹⁾ Yankov D. *et al.* Disk aware discord discovery: finding unusual time series in terabyte sized datasets. ICDM 2007. pp. 381-390. DOI: [10.1109/ICDM.2007.61](https://doi.org/10.1109/ICDM.2007.61).

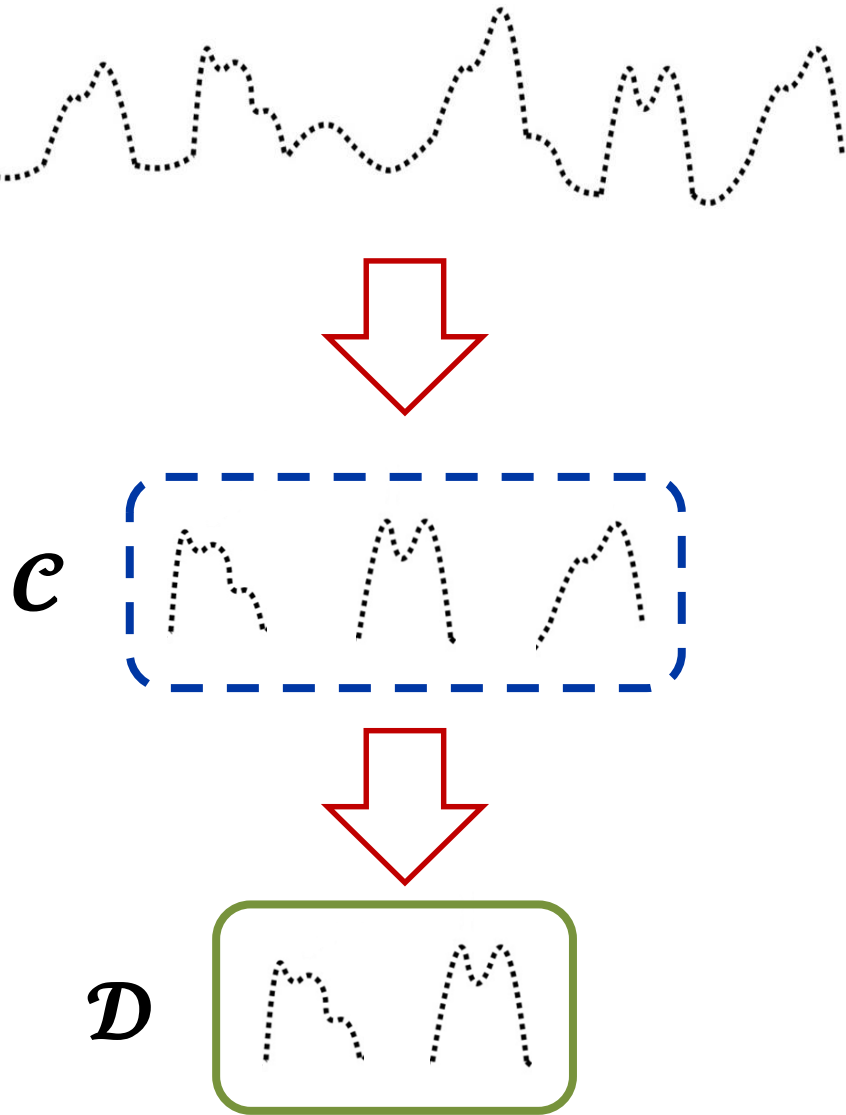
²⁾ Nakamura T. *et al.* MERLIN: parameter-free discovery of arbitrary length anomalies in massive time series archives. ICDM 2020. pp. 1190-1195. DOI: [10.1109/ICDM50108.2020.00147](https://doi.org/10.1109/ICDM50108.2020.00147).

³⁾ Kraeva Y., Zymbler M. A parallel discord discovery algorithm for a graphics processor. PRIA. 2023. 33(2). pp. 101-112. DOI: [10.1134/S1054661823020062](https://doi.org/10.1134/S1054661823020062).

⁴⁾ Zymbler M., Kraeva Y. High-performance time series anomaly discovery on graphics processors. Mathematics. 2023. 11(14). art. 3193. DOI: [10.3390/math11143193](https://doi.org/10.3390/math11143193).

⁵⁾ Kraeva Y., Zymbler M. Anomaly detection in long time series on high-performance cluster with GPUs. Numerical Methods and Programming. 2023. 24(3). pp. 291-304. DOI: [10.26089/NumMet.v24r320](https://doi.org/10.26089/NumMet.v24r320).

Discovery of fixed-length discords (DRAG algorithm)



1. Selection

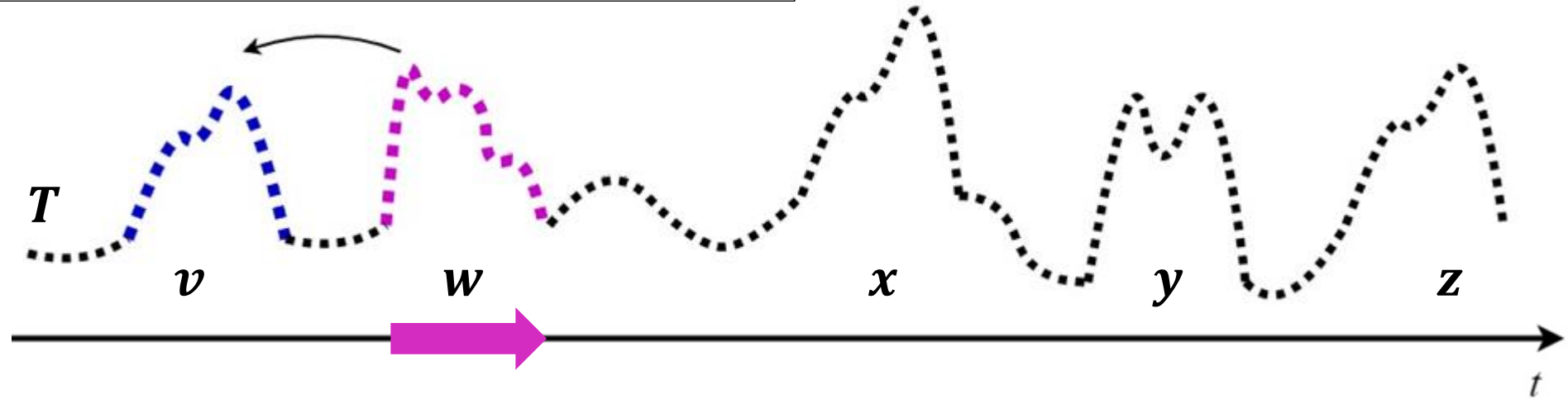
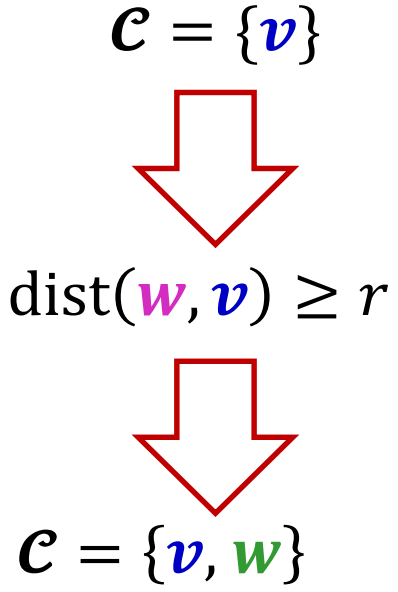
Through one full scan of the time series, create a **set of candidates** to discords

2. Refinement

Through one full scan of the time series, **prune false positive** candidates

Candidate selection

```
C := {T1,m}  
while not end of T  
  get next subsequence s  
  isCandidate := TRUE  
  for each ci ∈ C and s ∩ ci = ∅  
    if dist(s, ci) < r then  
      C := C \ ci; isCandidate := FALSE  
  if isCandidate = TRUE then C := C ∪ s
```



Candidate selection

```
 $\mathcal{C} := \{T_{1,m}\}$   
while not end of  $T$   
  get next subsequence  $s$   
   $isCandidate := \text{TRUE}$   
  for each  $c_i \in \mathcal{C}$  and  $s \cap c_i = \emptyset$   
    if  $\text{dist}(s, c_i) < r$  then  
       $\mathcal{C} := \mathcal{C} \setminus c_i$ ;  $isCandidate := \text{FALSE}$   
  if  $isCandidate = \text{TRUE}$  then  $\mathcal{C} := \mathcal{C} \cup s$ 
```

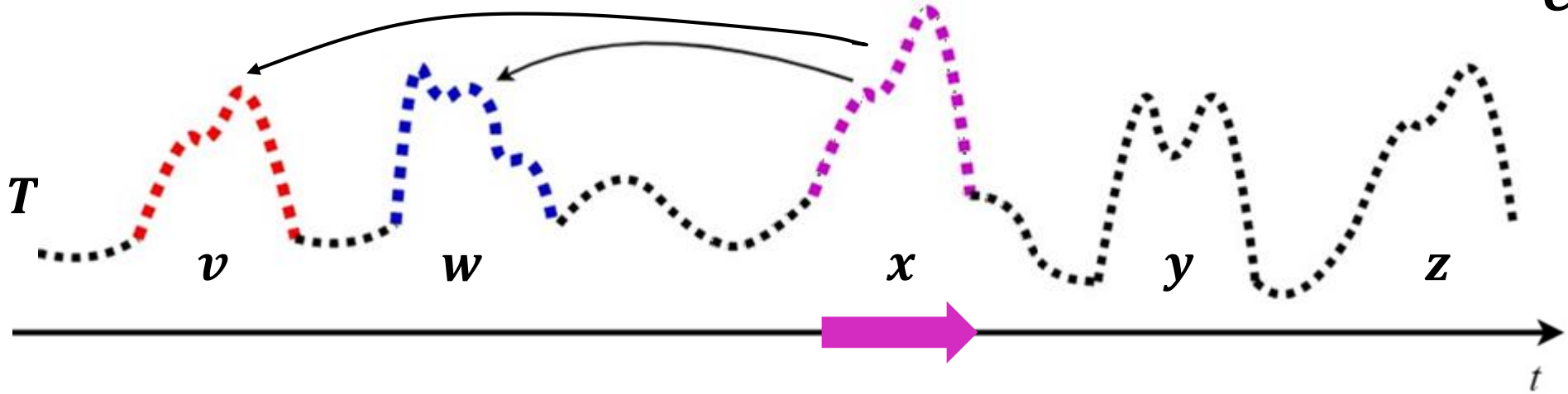
$\mathcal{C} = \{v, w\}$

↓

$\text{dist}(x, v) < r$
 $\text{dist}(x, w) \geq r$

↓

$\mathcal{C} = \{w\}$



Candidate selection

$\mathcal{C} := \{T_{1,m}\}$

while not end of T

 get next subsequence s

$isCandidate := \text{TRUE}$

for each $c_i \in \mathcal{C}$ **and** $s \cap c_i = \emptyset$

if $\text{dist}(s, c_i) < r$ **then**

$\mathcal{C} := \mathcal{C} \setminus c_i$; $isCandidate := \text{FALSE}$

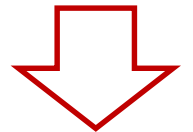
if $isCandidate = \text{TRUE}$ **then** $\mathcal{C} := \mathcal{C} \cup s$

$\mathcal{C} = \{w, y\}$

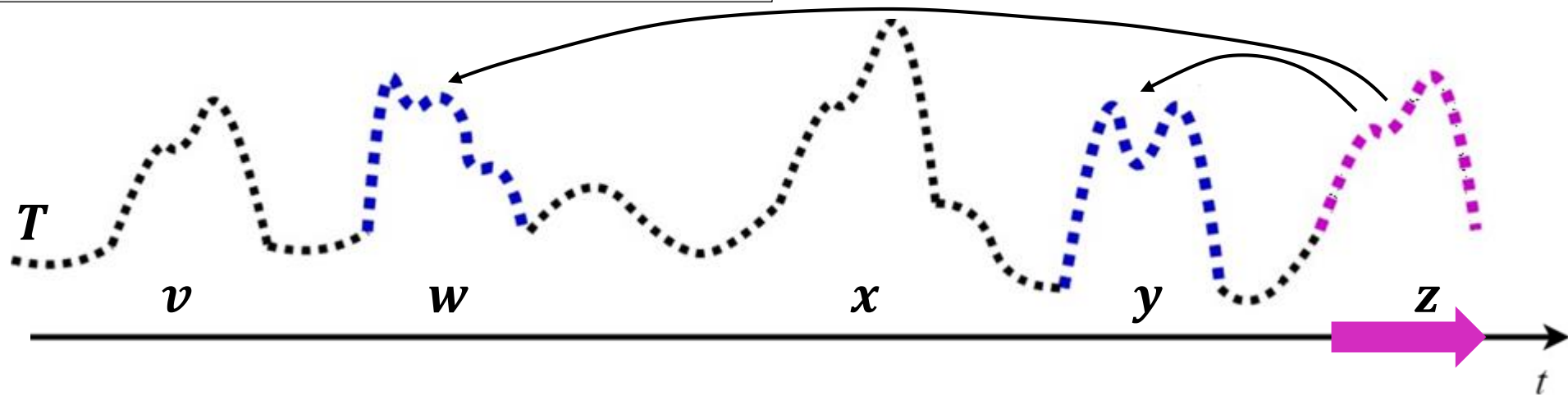


$\text{dist}(z, w) \geq r$

$\text{dist}(z, y) \geq r$



$\mathcal{C} = \{w, y, z\}$



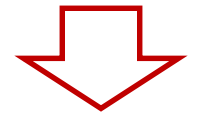
Discord refinement

```
 $\mathcal{D} := \mathcal{C}$   
while not end of  $T$   
  get next subsequence  $s$   
  for each  $d_i \in \mathcal{D}$  and  $s \cap d_i = \emptyset$   
    if  $\text{dist}(s, d_i) < r$  then  
       $\mathcal{D} := \mathcal{D} \setminus d_i$ 
```

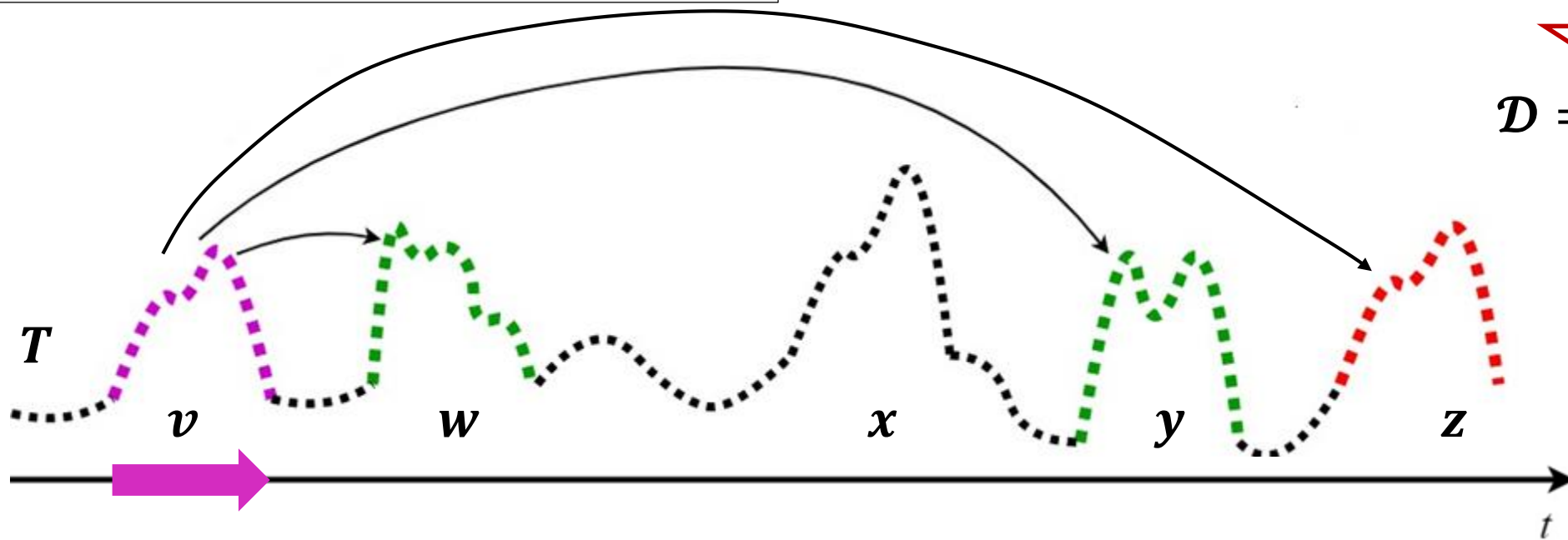
$$\mathcal{D} = \{w, y, z\}$$



$$\begin{aligned} \text{dist}(v, w) &\geq r \\ \text{dist}(v, y) &\geq r \\ \text{dist}(v, z) &< r \end{aligned}$$



$$\mathcal{D} = \{w, y\}$$



Discord refinement

$\mathcal{D} := \mathcal{C}$

while not end of T

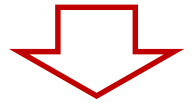
 get next subsequence s

 for each $d_i \in \mathcal{D}$ and $s \cap d_i = \emptyset$

 if $\text{dist}(s, d_i) < r$ then

$\mathcal{D} := \mathcal{D} \setminus d_i$

$\mathcal{D} = \{w, y\}$

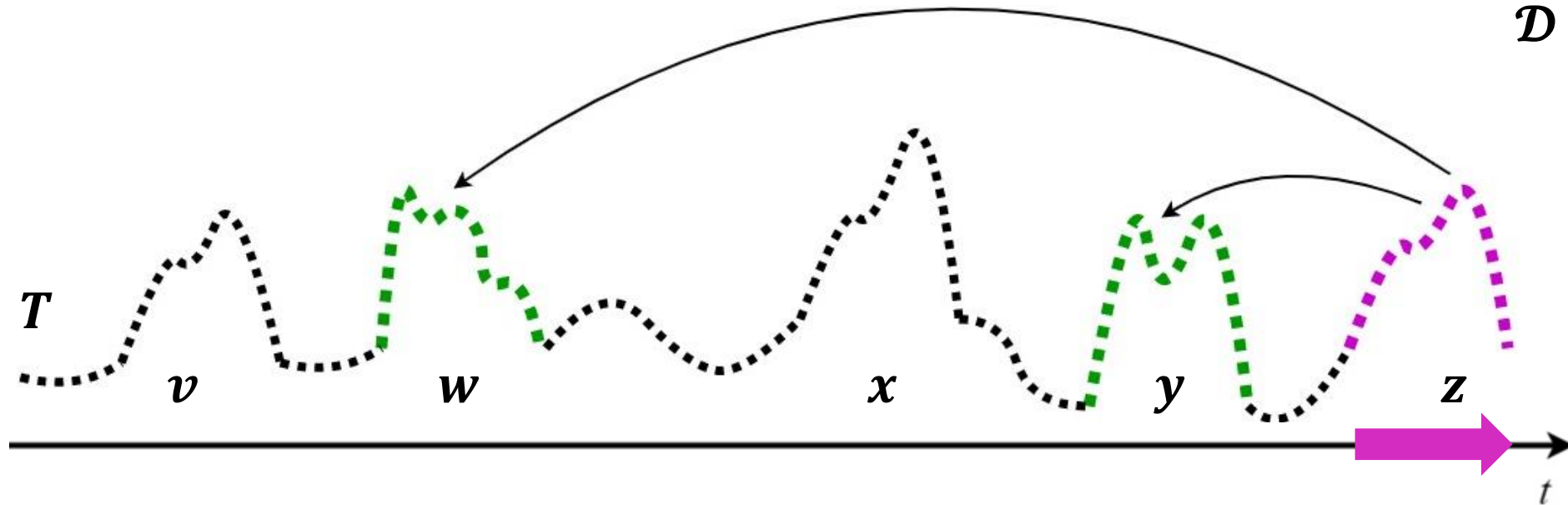


$\text{dist}(z, w) \geq r$

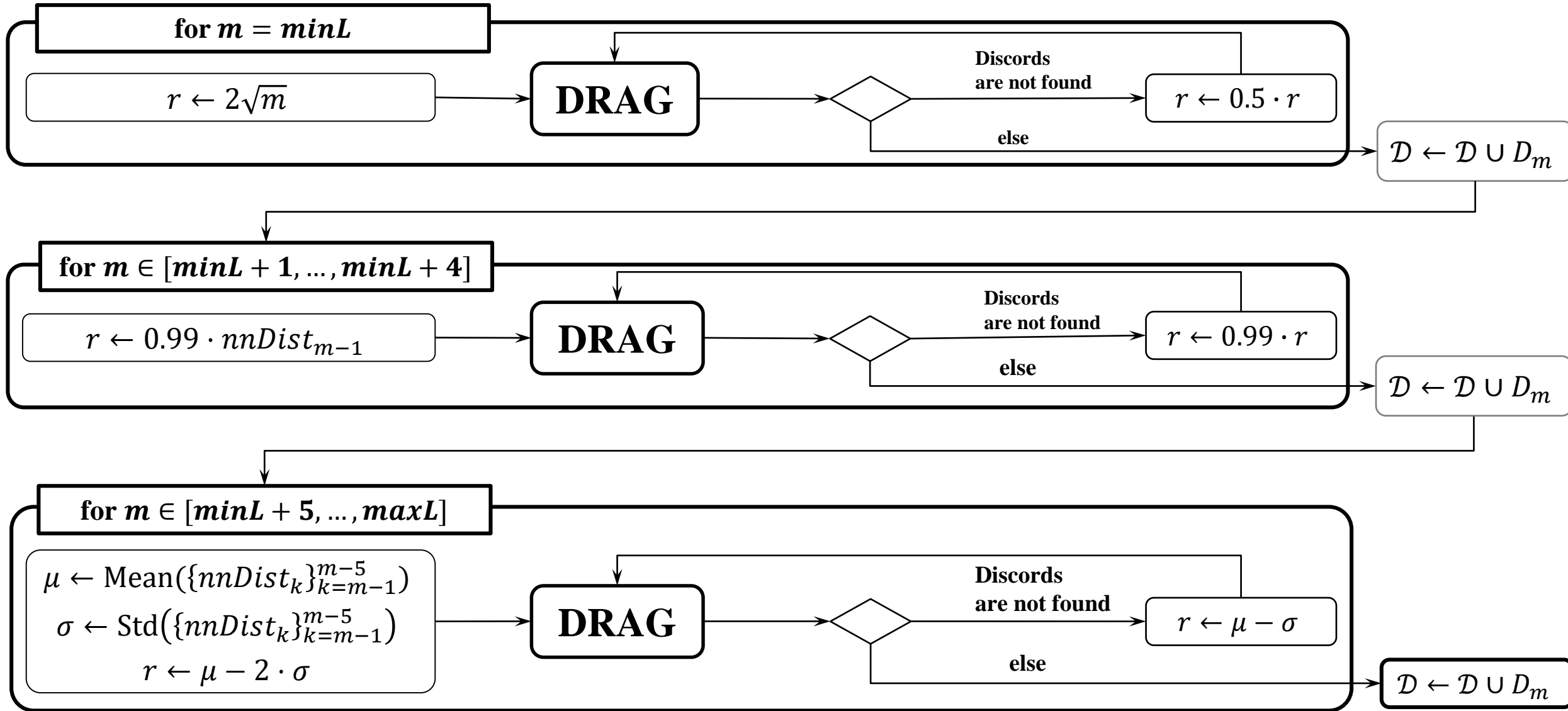
$\text{dist}(z, y) \geq r$



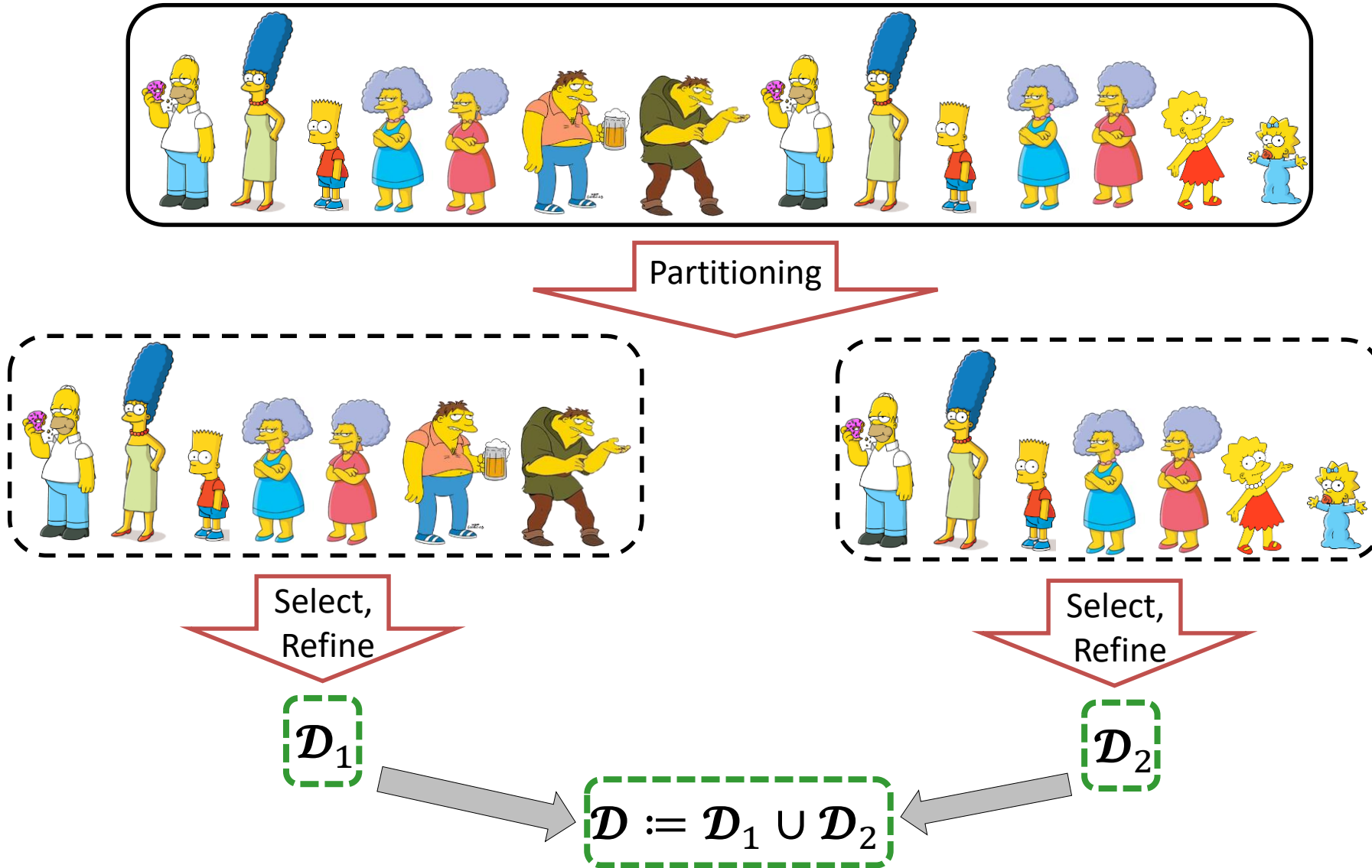
$\mathcal{D} = \{w, y\}$



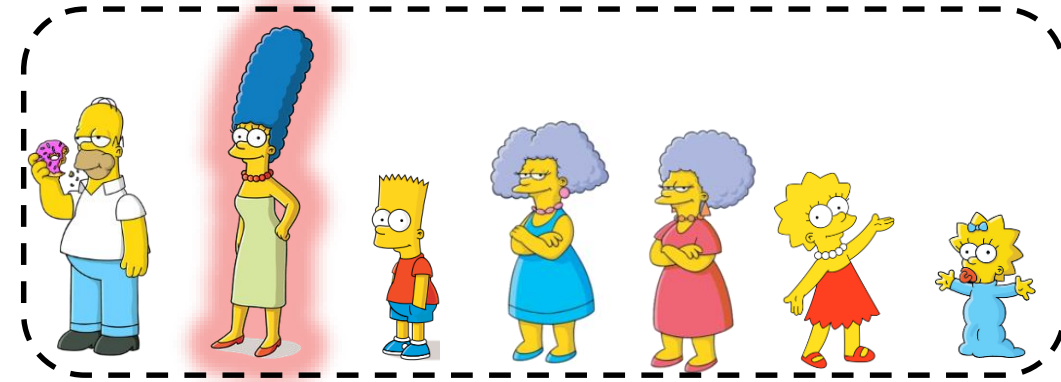
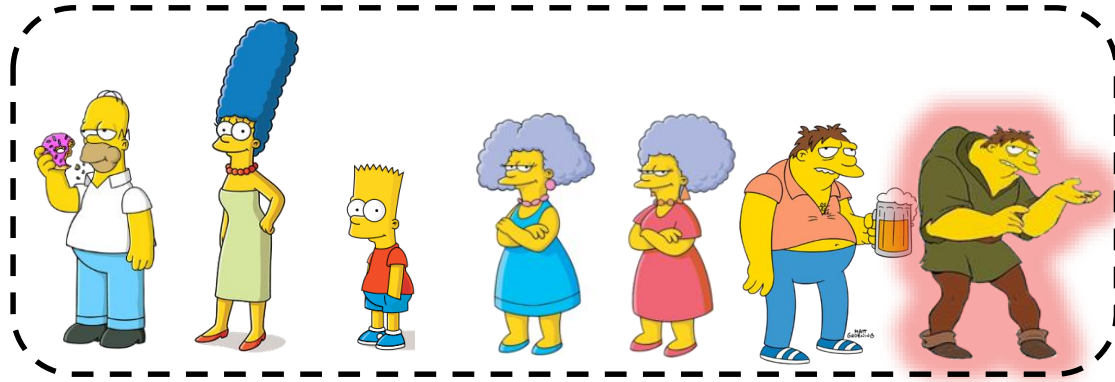
Discovery of all-length discords (MERLIN algorithm)



Can we discover discords in parallel?



Naïve parallel discord discovery does not work

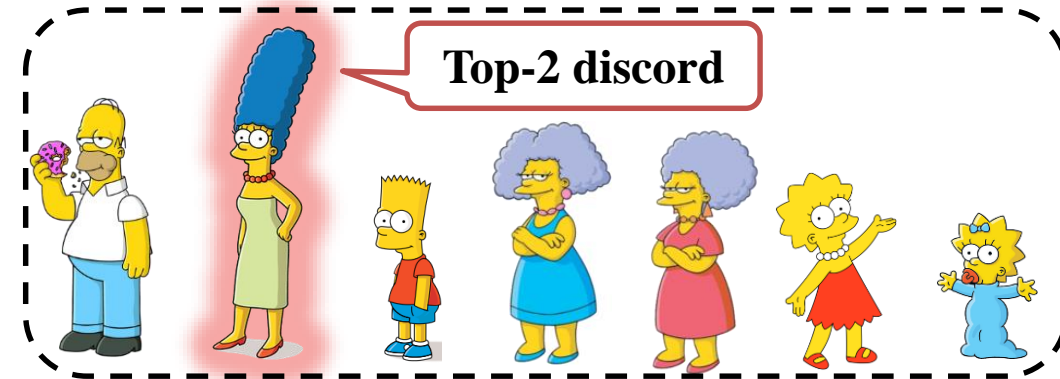
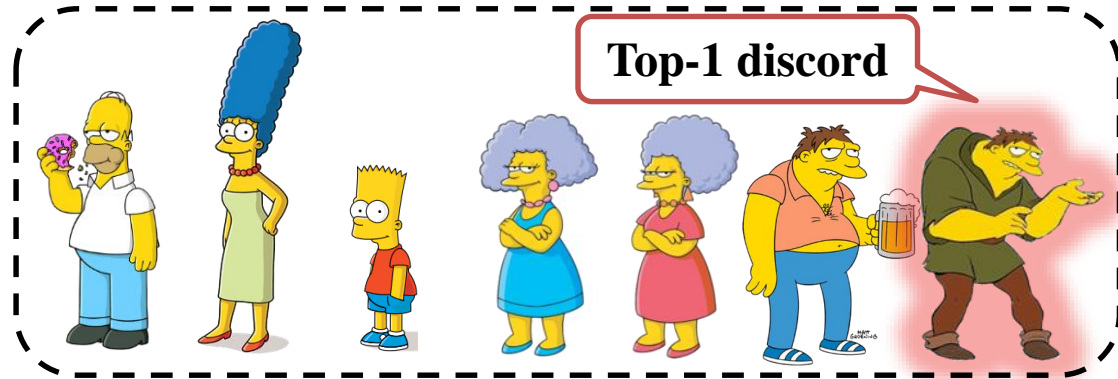


0	5	2	4	4	6	8
5	0	2.5	3	3	6	10
2	2.5	0	4	4	6	9
4	3	4	0	0.5	5	8
4	3	4	0.5	0	5	8
6	6	6	5	5	0	7
8	10	9	8	8	7	0



0	5	2	4	4	2.5	2.5
5	0	2.5	3	3	2	2
2	2.5	0	4	4	0.5	0.5
4	3	4	0	0.5	4	4
4	3	4	0.5	0	4	4
2.5	2	0.5	4	4	0	0.5
2.5	2	0.5	4	4	0.5	0

Naïve parallel discord discovery does not work

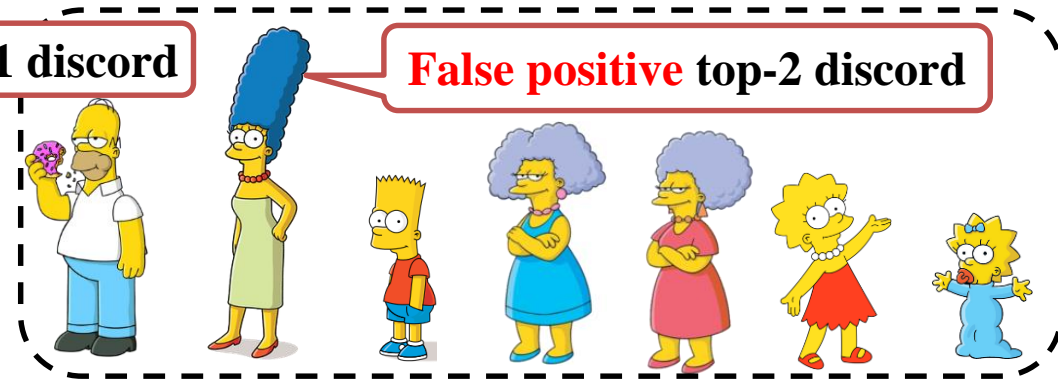
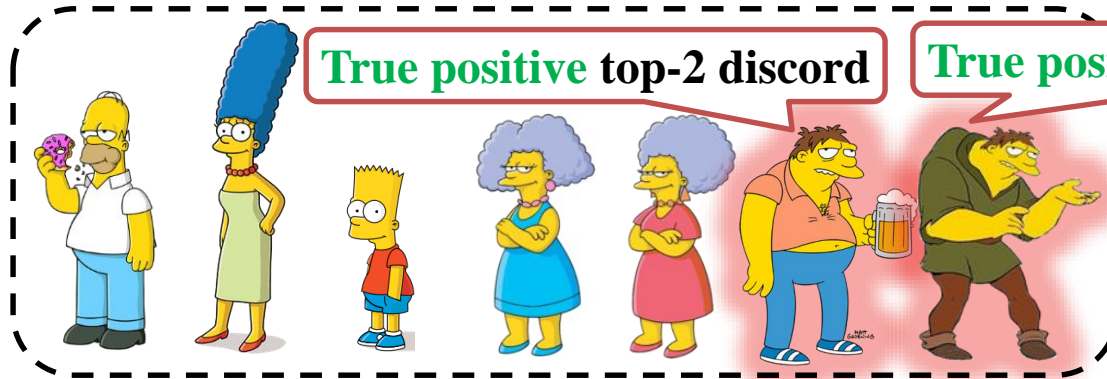


0	5	2	4	4	6	8
5	0	2.5	3	3	6	10
2	2.5	0	4	4	6	9
4	3	4	0	0.5	5	8
4	3	4	0.5	0	5	8
6	6	6	5	5	0	7
8	10	9	8	8	7	0



0	5	2	4	4	2.5	2.5
5	0	2.5	3	3	2	2
2	2.5	0	4	4	0.5	0.5
4	3	4	0	0.5	4	4
4	3	4	0.5	0	4	4
2.5	2	0.5	4	4	0	0.5
2.5	2	0.5	4	4	0.5	0

Naïve parallel discord discovery does not work

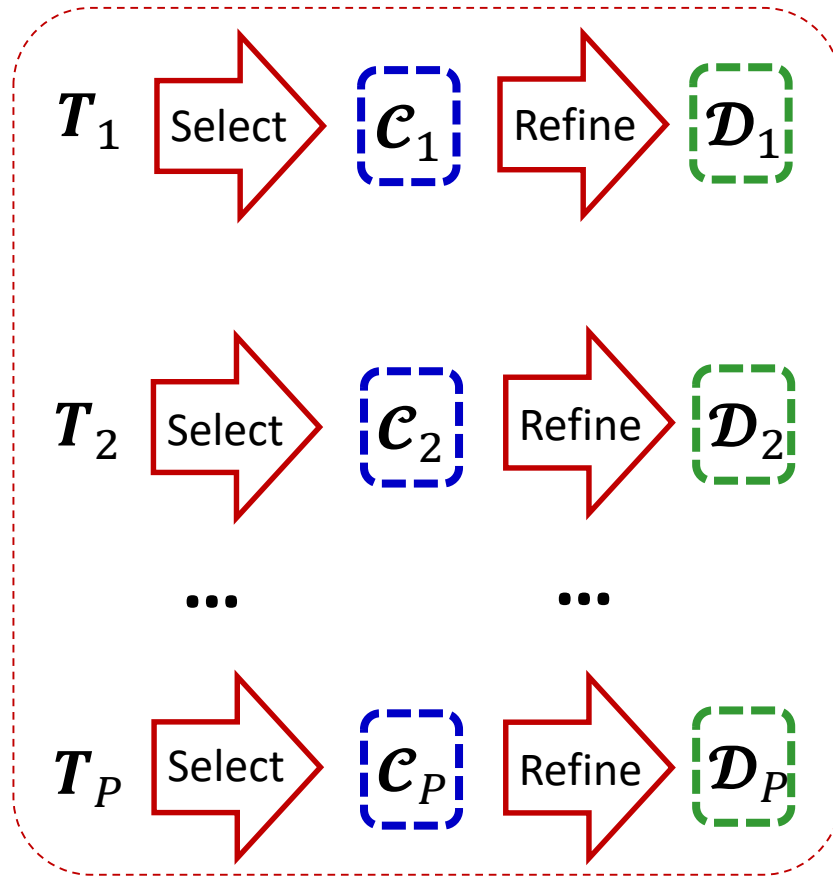
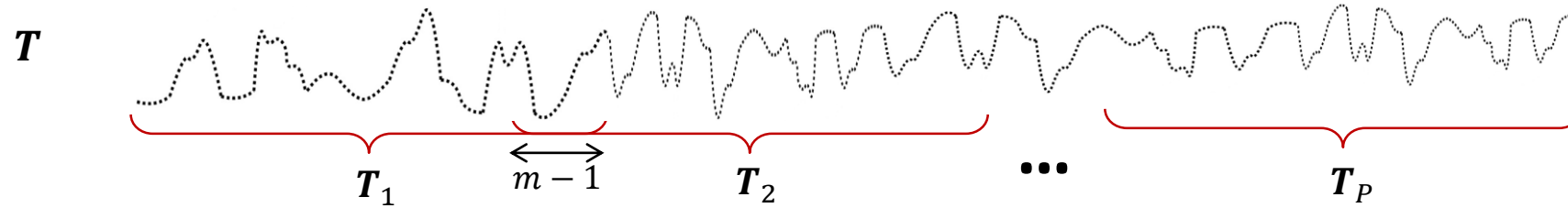


0	5	2	4	4	6	8
5	0	2.5	3	3	6	10
2	2.5	0	4	4	6	9
4	3	4	0	0.5	5	8
4	3	4	0.5	0	5	8
6	6	6	5	5	0	7
8	10	9	8	8	7	0



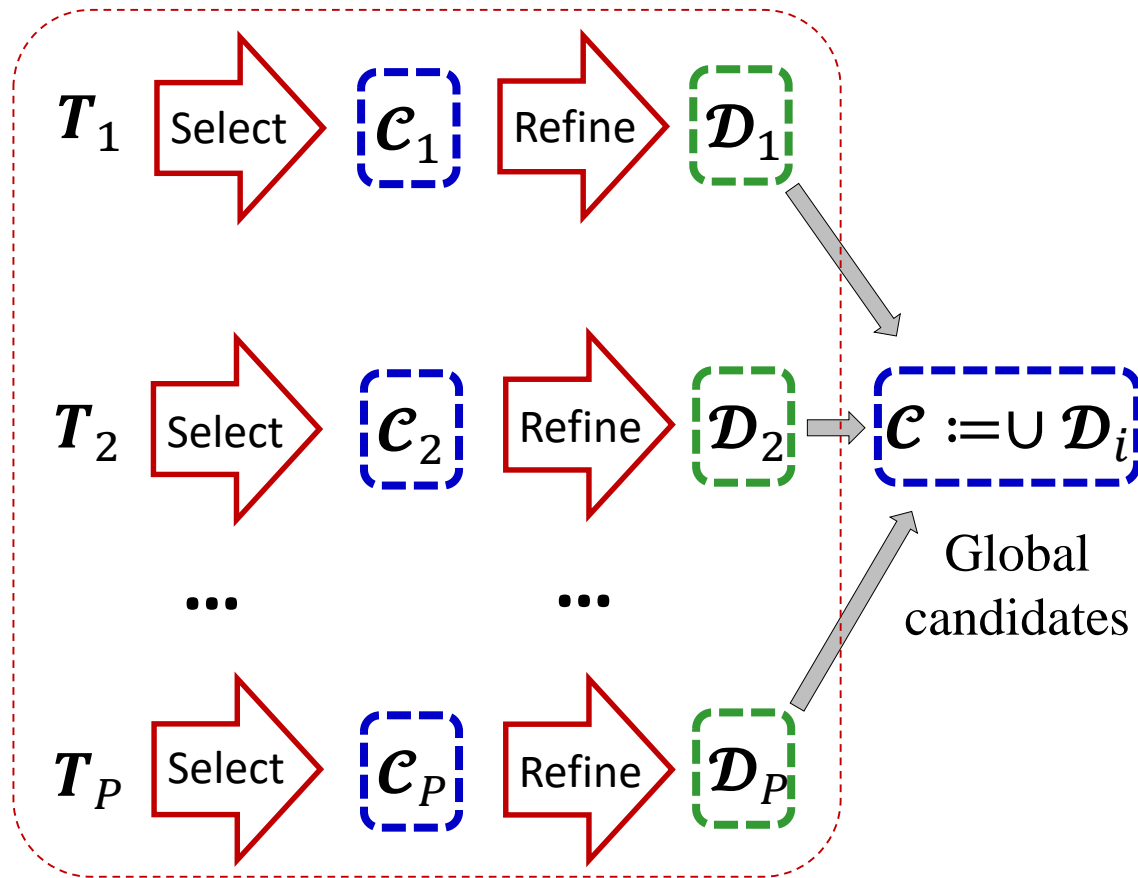
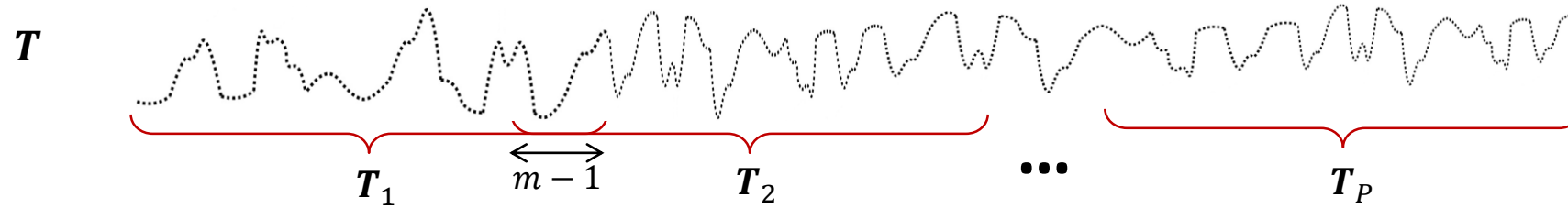
0	5	2	4	4	2.5	2.5
5	0	2.5	3	3	2	2
2	2.5	0	4	4	0.5	0.5
4	3	4	0	0.5	4	4
4	3	4	0.5	0	4	4
2.5	2	0.5	4	4	0	0.5
2.5	2	0.5	4	4	0.5	0

Parallel discovery: Local discords must be globally refined



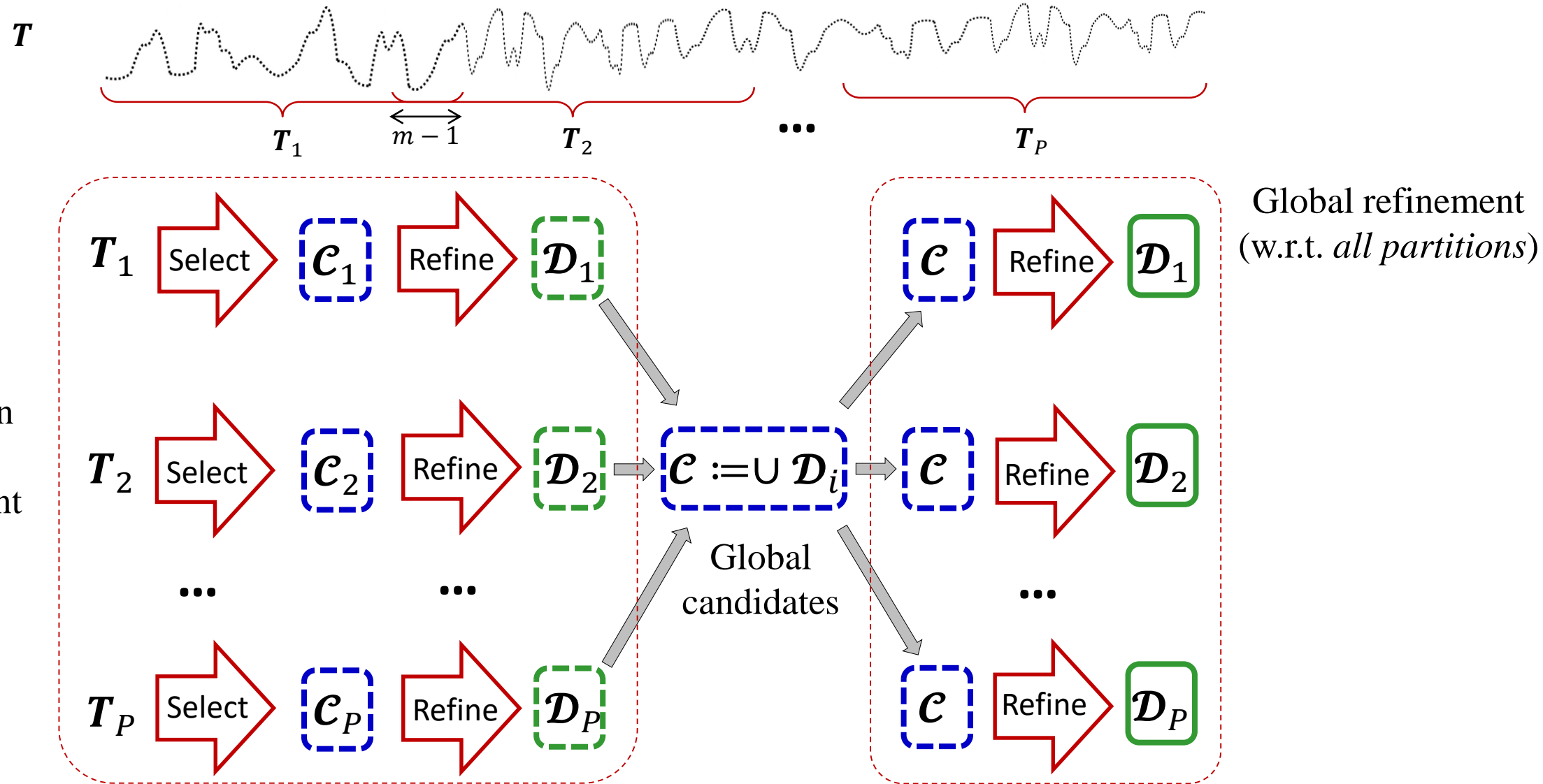
Local selection
and
local refinement
(w.r.t. *each*
partition)

Parallel discovery: Local discords must be globally refined

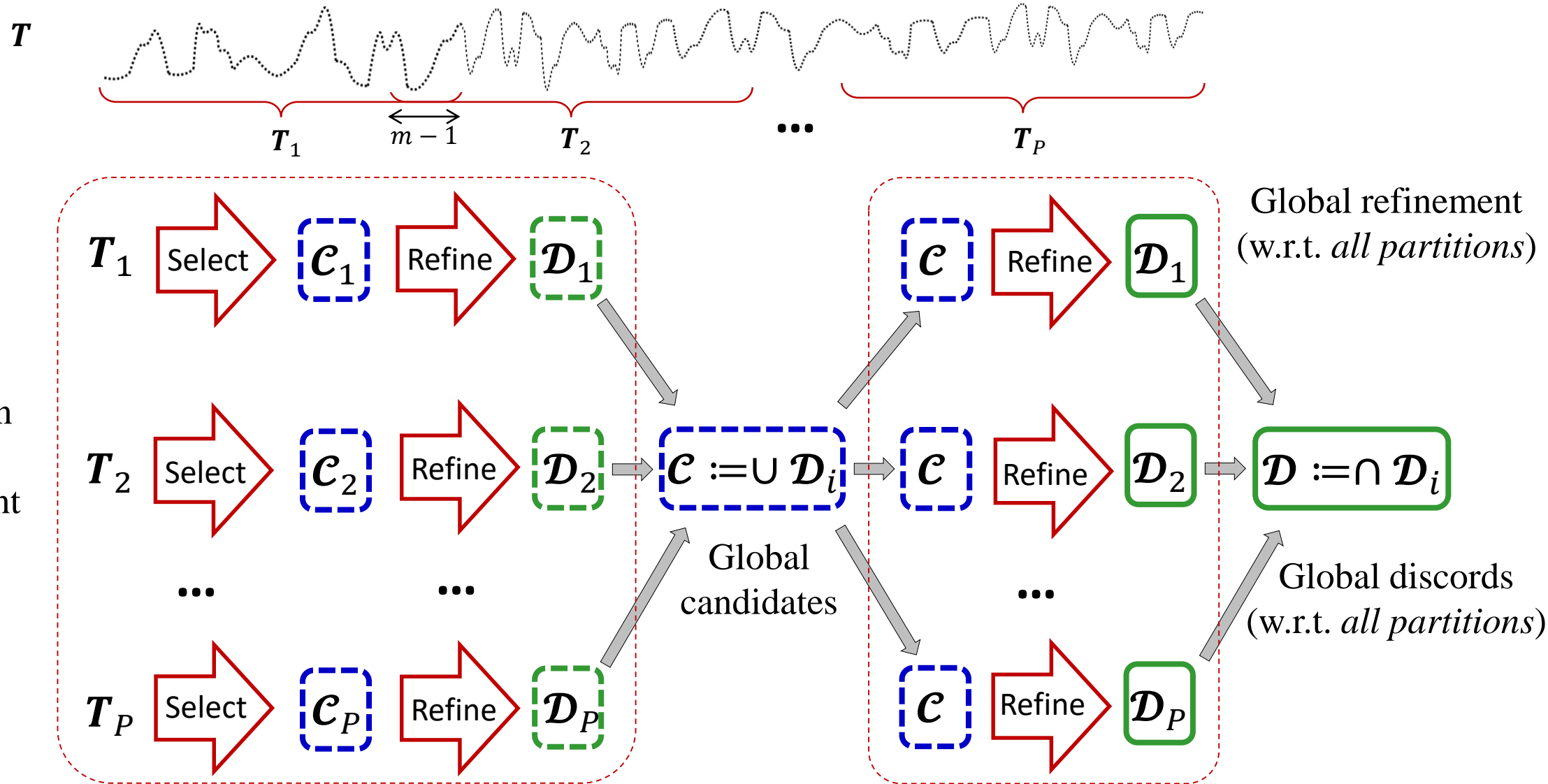


Local selection
and
local refinement
(w.r.t. *each*
partition)

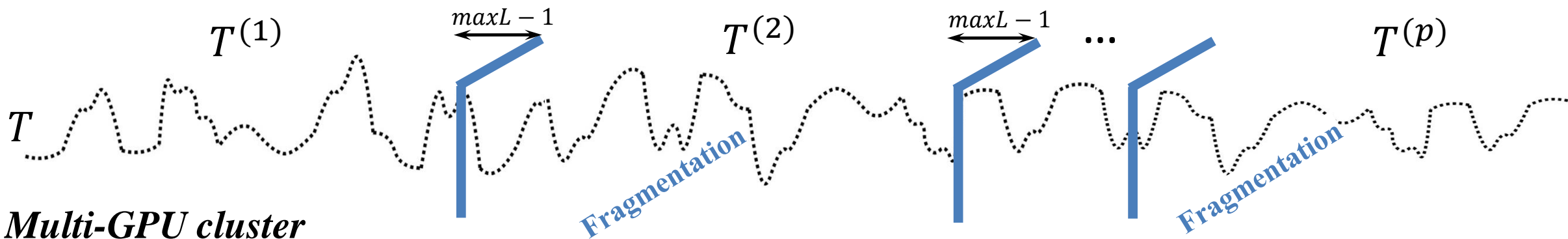
Parallel discovery: Local discords must be globally refined



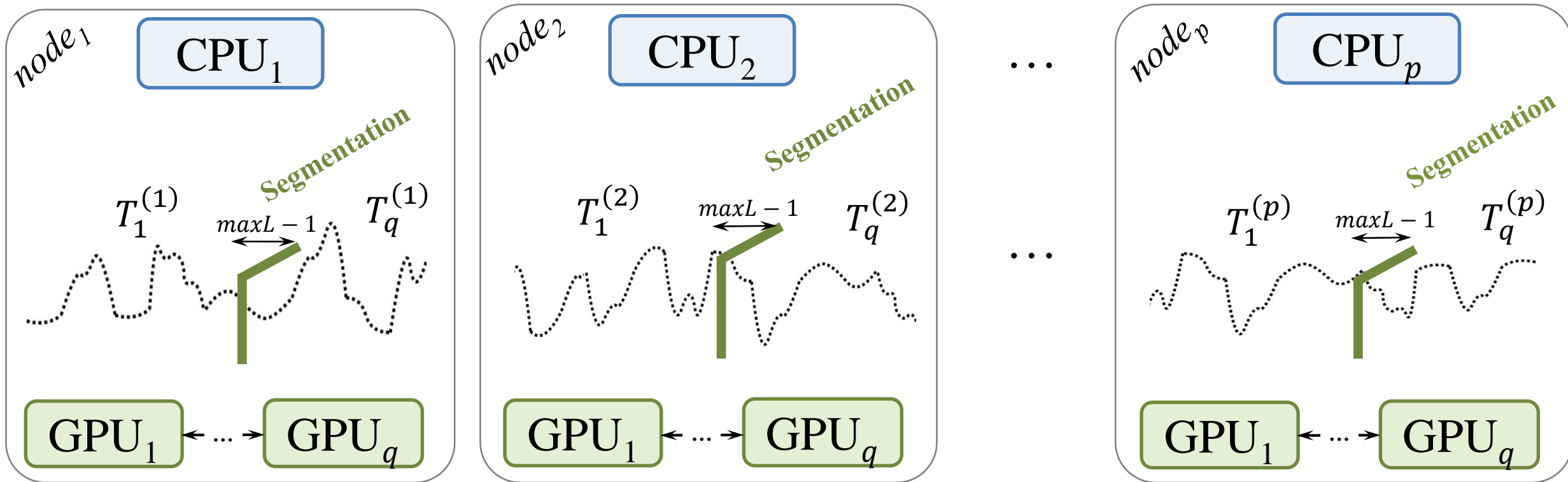
Parallel discovery: Local discords must be globally refined



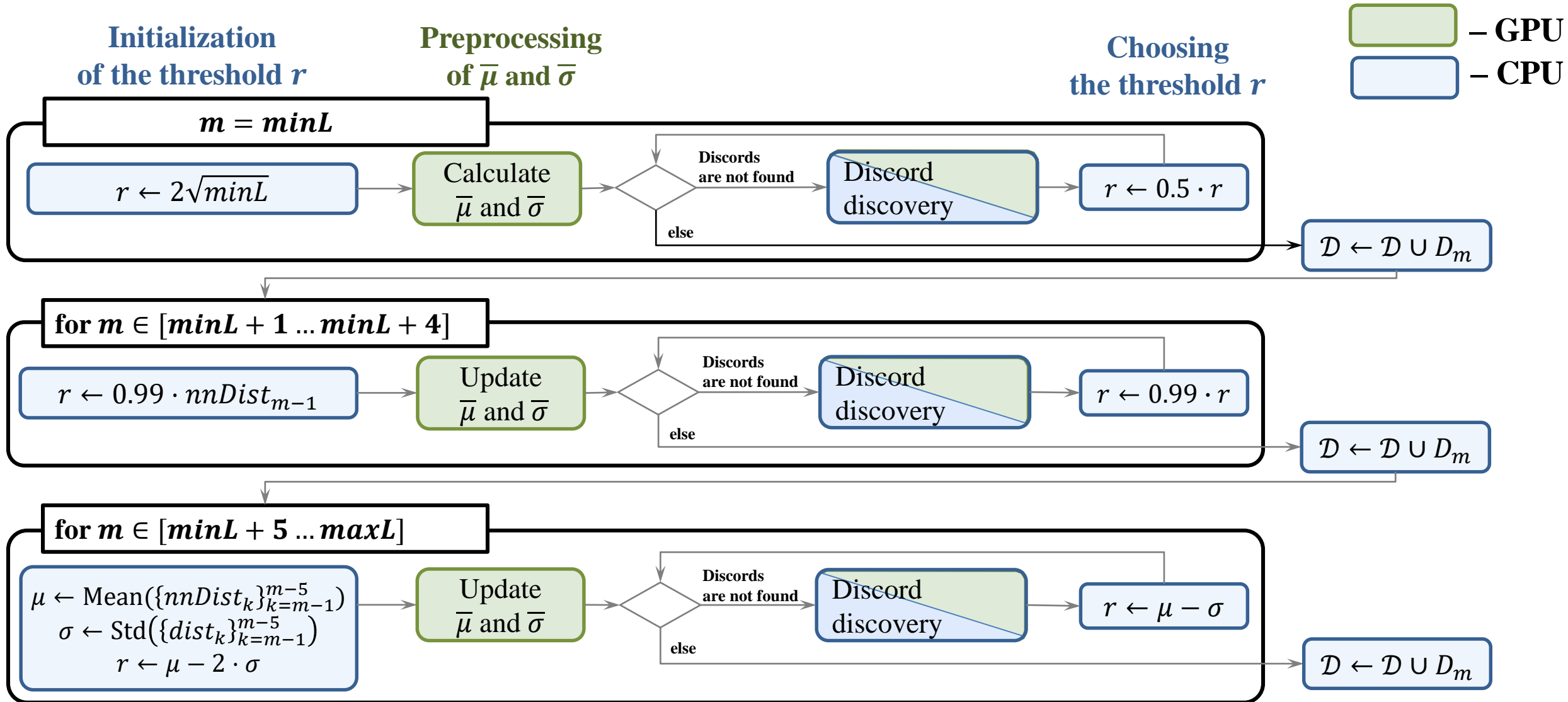
Two-level data parallelism (PADDi algorithm)



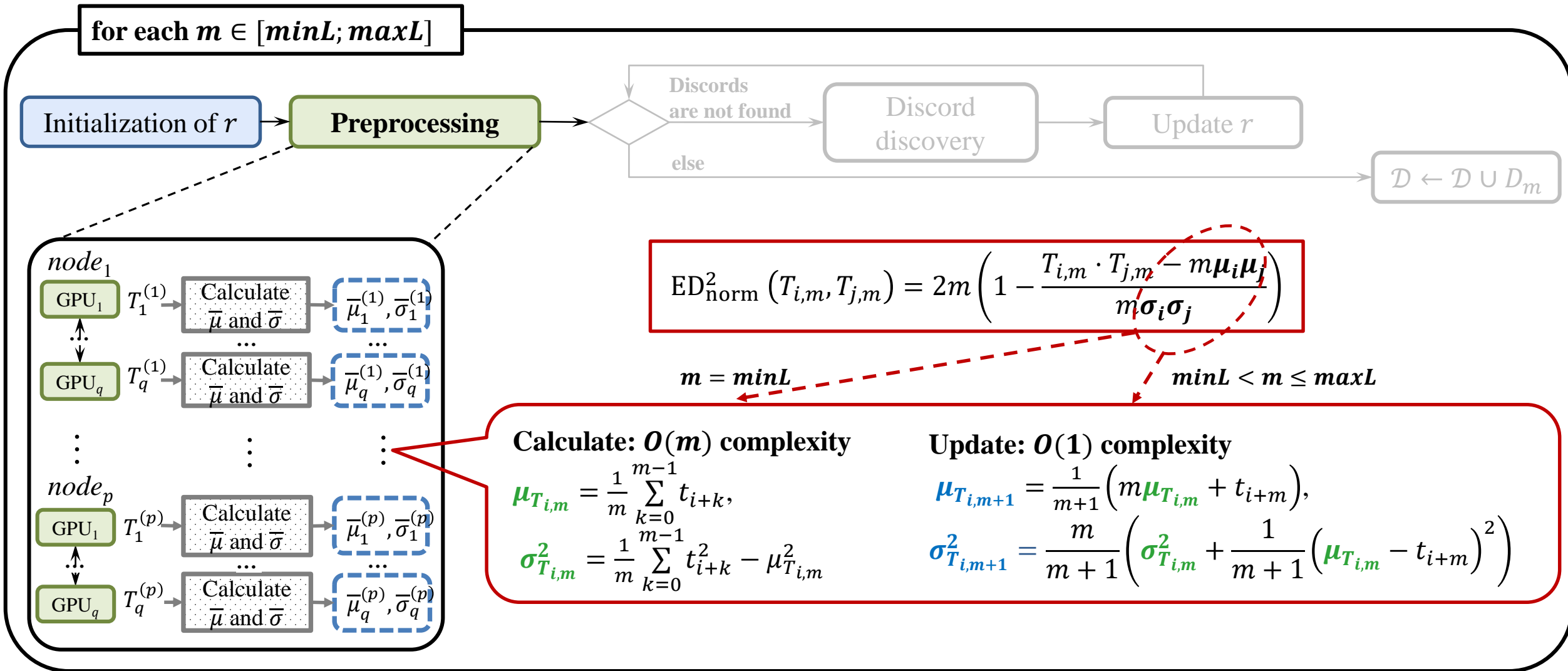
Multi-GPU cluster



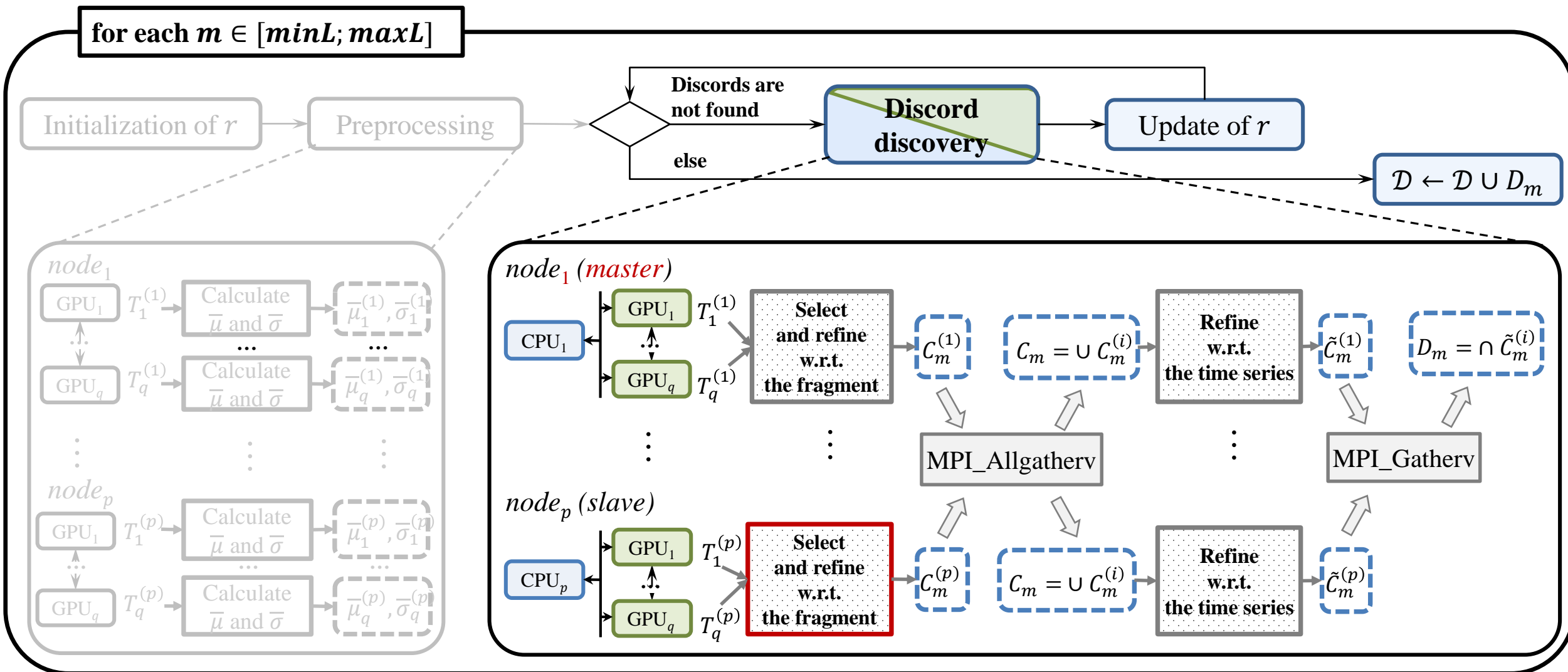
General scheme of calculations



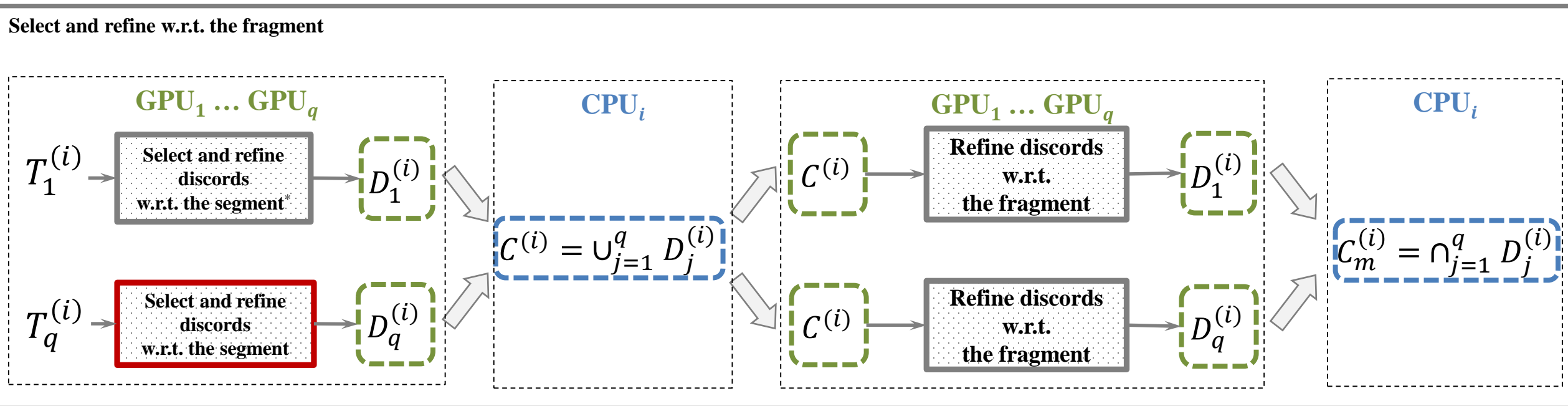
Preprocessing



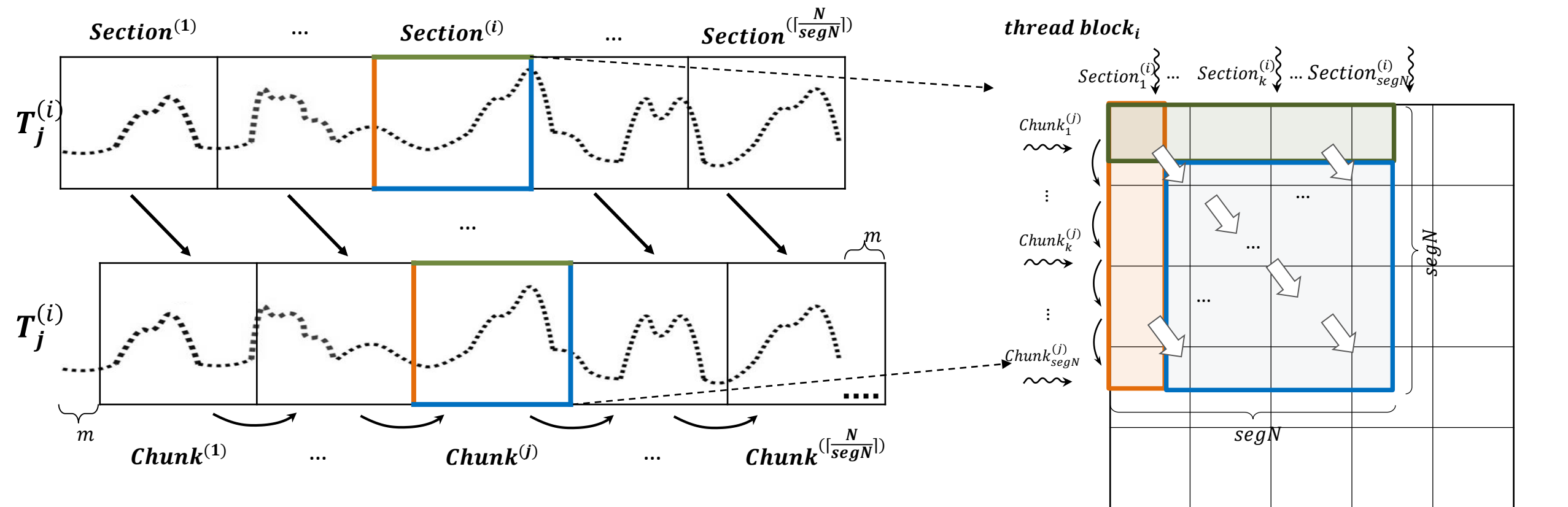
Discord discovery



Candidate selection and discord refinement w.r.t. the fragment



Candidate selection and discord refinement w.r.t. the segment



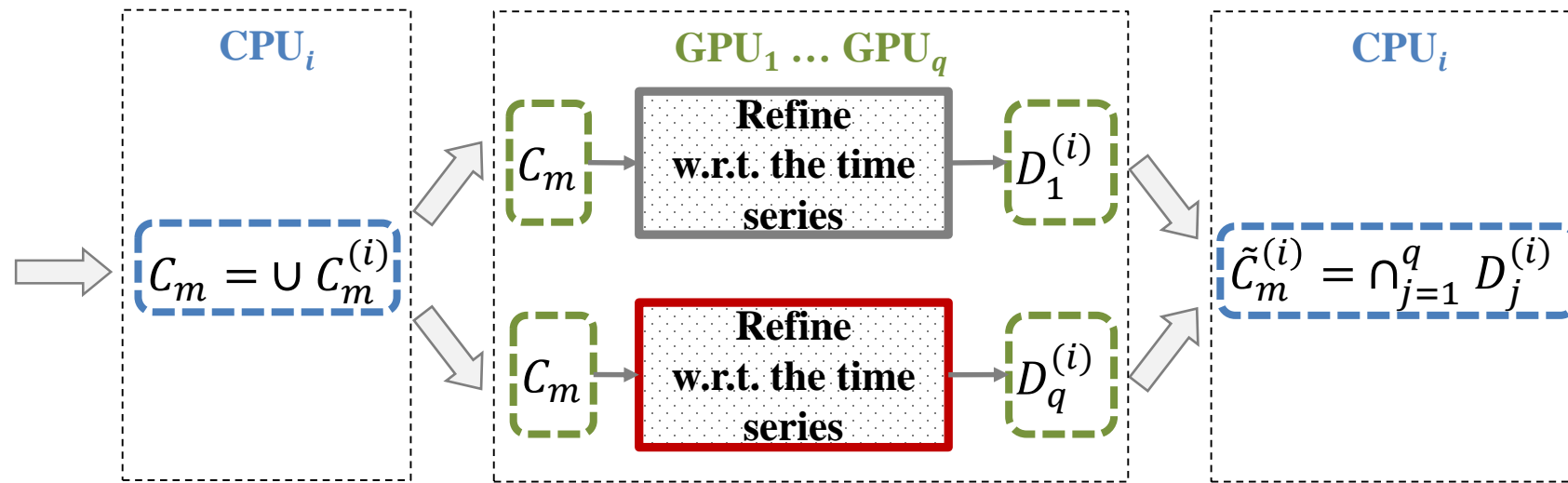
$$ED_{\text{norm}}^2(T_{i,m}, T_{j,m}) = 2m \left(1 - \frac{T_{i,m} \cdot T_{j,m} - m\mu_i\mu_j}{m\sigma_i\sigma_j} \right)$$

$$\text{QTrow}^{(i)}(tid) = \sum_{k=1}^m T_{tid}^{(i)}(k) \cdot \text{Chunk}_1^{(j)}(k) \quad \text{QTcol}^{(i)}(tid) = \sum_{k=1}^m T_1^{(i)}(k) \cdot \text{Chunk}_{tid}^{(j)}(k)$$

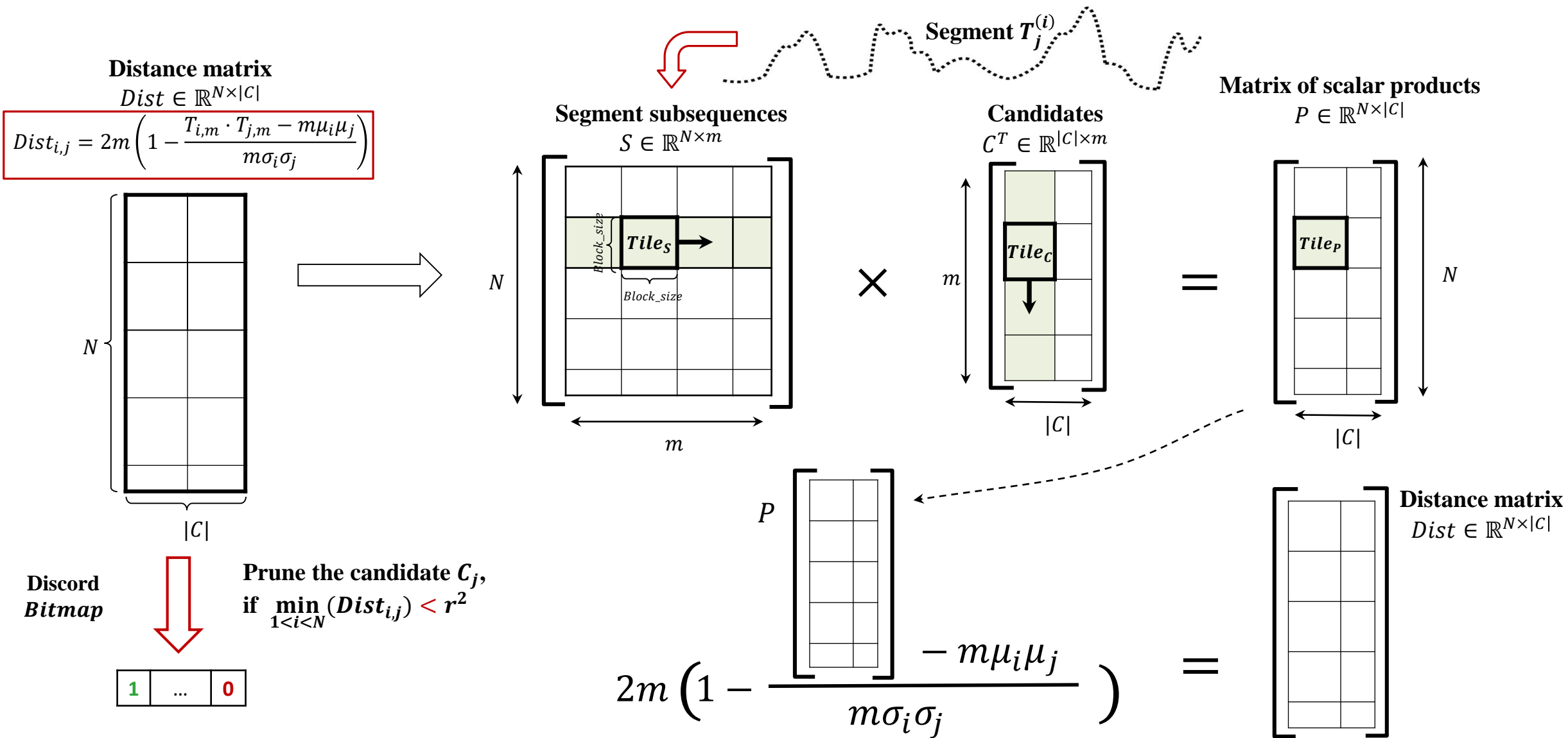
$O(1)$ instead of $O(m)$!

$$\text{QTrow}^{(i)}(tid) = \text{QTrow}^{(i)}(tid - 1) - T_{tid-1}^{(i)}(1) \cdot \text{Chunk}_{tid-1}^{(j)}(1) + T_{tid}^{(i)}(m) \cdot \text{Chunk}_{tid}^{(j)}(m)$$

Discord refinement w.r.t. the time series



Discord refinement



Experimental evaluation

- **HPC cluster Lobachevsky (University of Nizhny Novgorod)**
64 × (2×Sandy Bridge E5-2660 + 3×NVIDIA Kepler K20X (2 688 cores @0.732 GHz, 1.3 TFLOPS))
- **Data**

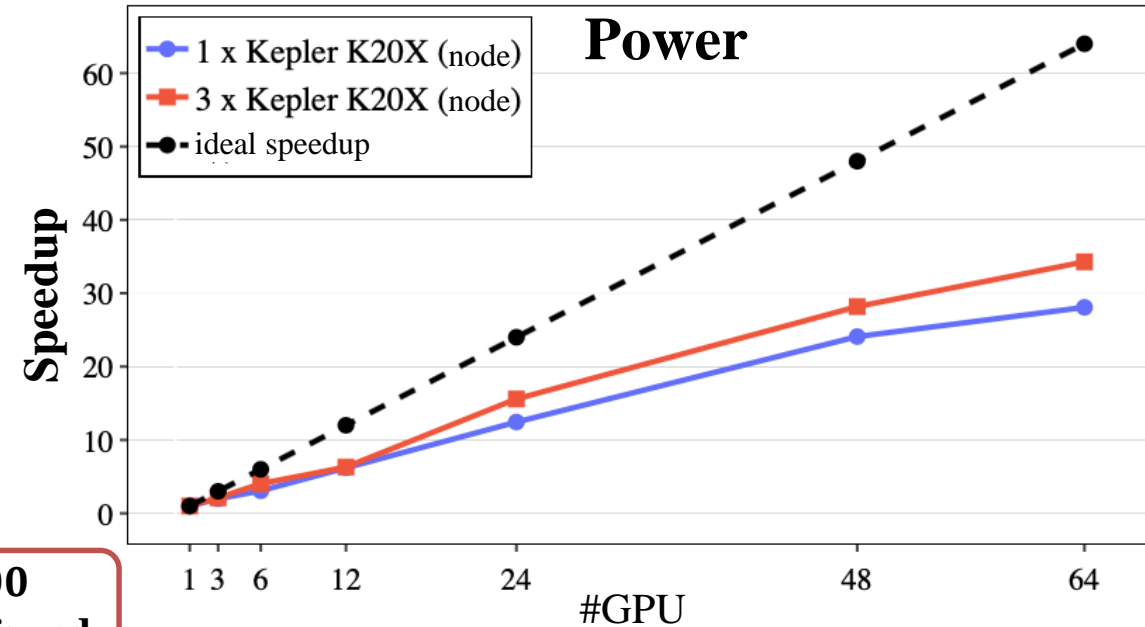
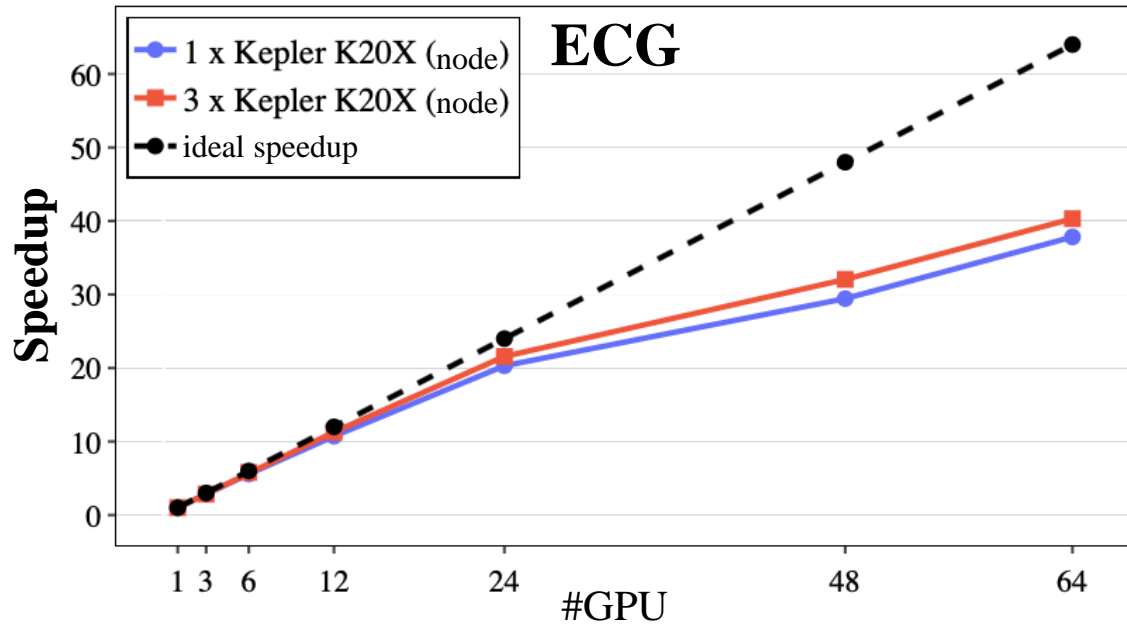
Time series	Length, n	Discord length range, $minL..maxL$	Subject domain
ECG ¹⁾	$2 \cdot 10^6$	64..128	ECG of an adult patient
Power ²⁾			Annual household power consumption

- **Metrics**
 - *Performance*: average running time over 10 launches (without I/O)
 - *Speedup*: $s(p) = t_1/t_p$, where t_1 and t_p are performance on one and p GPUs, respectively

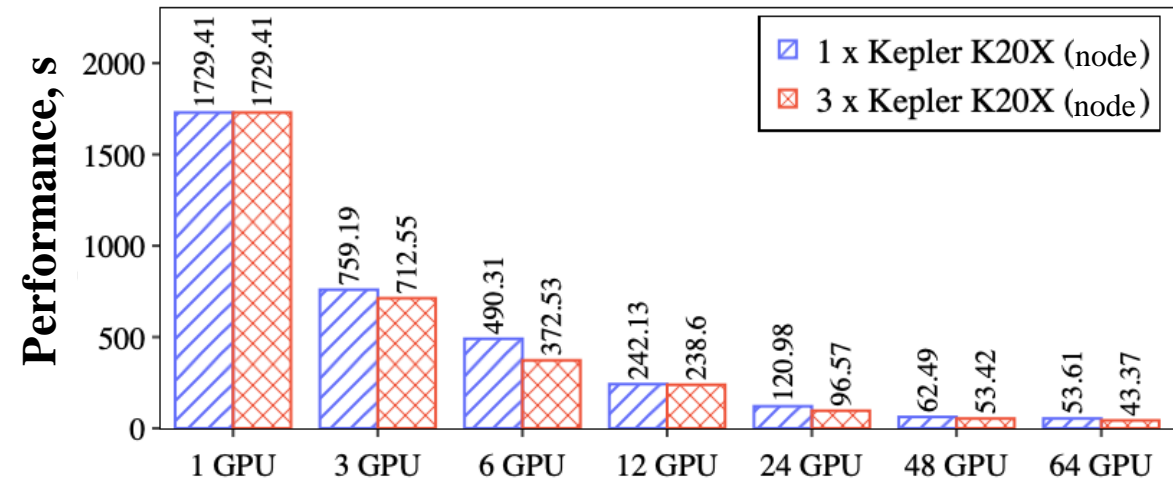
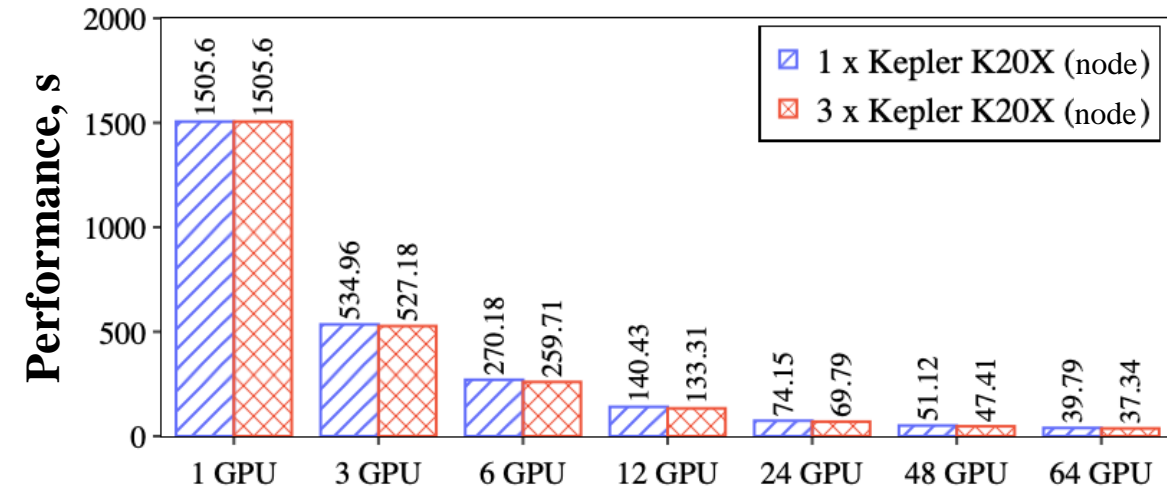
¹⁾Goldberger A. *et al.* PhysioBank, PhysioToolkit, and PhysioNet components of a new research resource for complex physiologic signals. Circulation. 2000. 101(23). pp. 215-220. DOI: [10.1161/01.CIR.101.23.e215](https://doi.org/10.1161/01.CIR.101.23.e215).

²⁾ Individual household electric power consumption. URL: <https://archive.ics.uci.edu/ml/datasets/individual+household+electric+power+consumption/>.

Speedup and performance

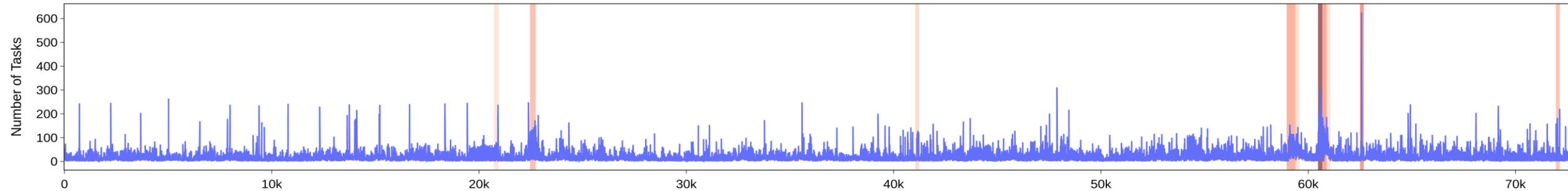


≈ 1500 discords found



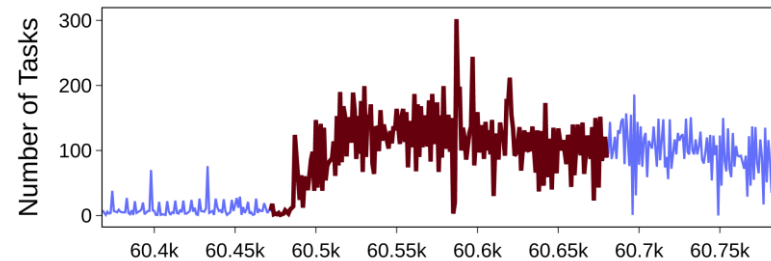
Case study: anomalies in HPC cluster workload

Tasks running in Alibaba PAI (Platform for Artificial Intelligence), July-August 2020*

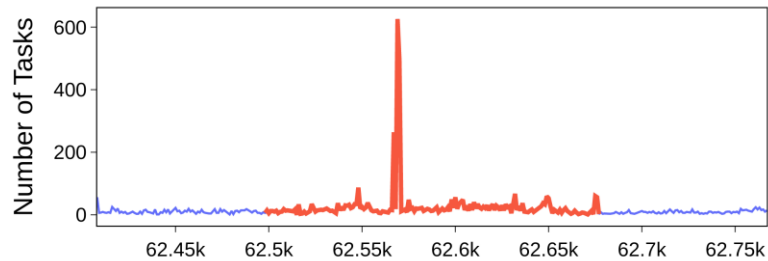


Top discords (range is 3-7 min)

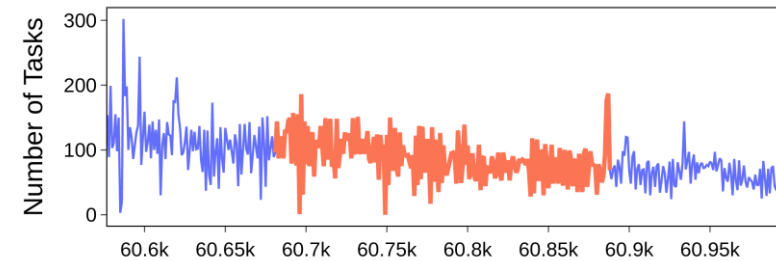
Top-1: $m = 209$ (3 min 29 sec)



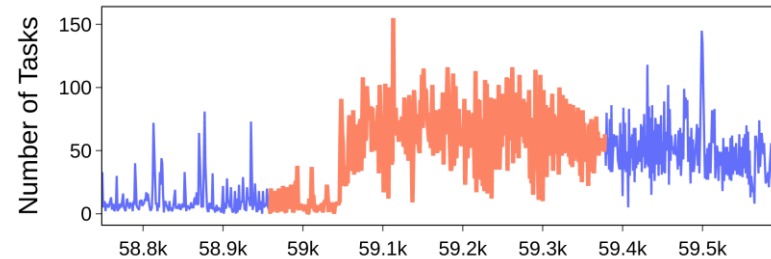
Top-2: $m = 180$ (3 min)



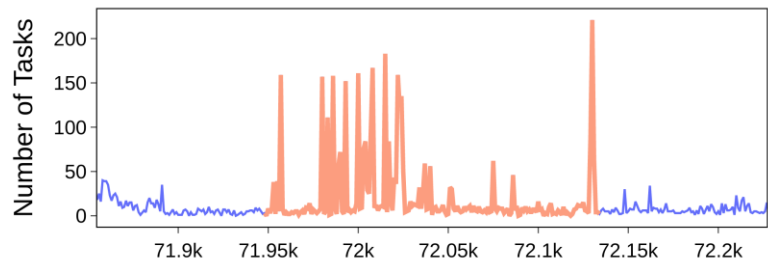
Top-3: $m = 208$ (3 min 28 sec)



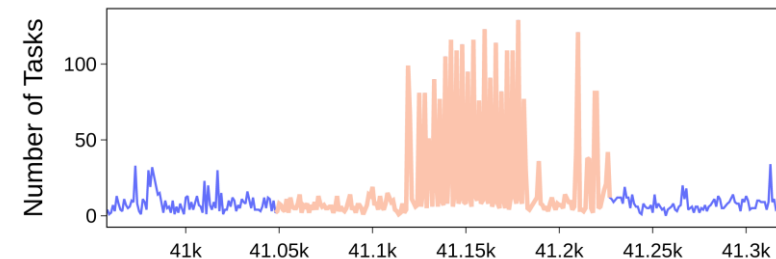
Top-4: $m = 420$ (7 min)



Top-5: $m = 283$ (4 min 43 sec)

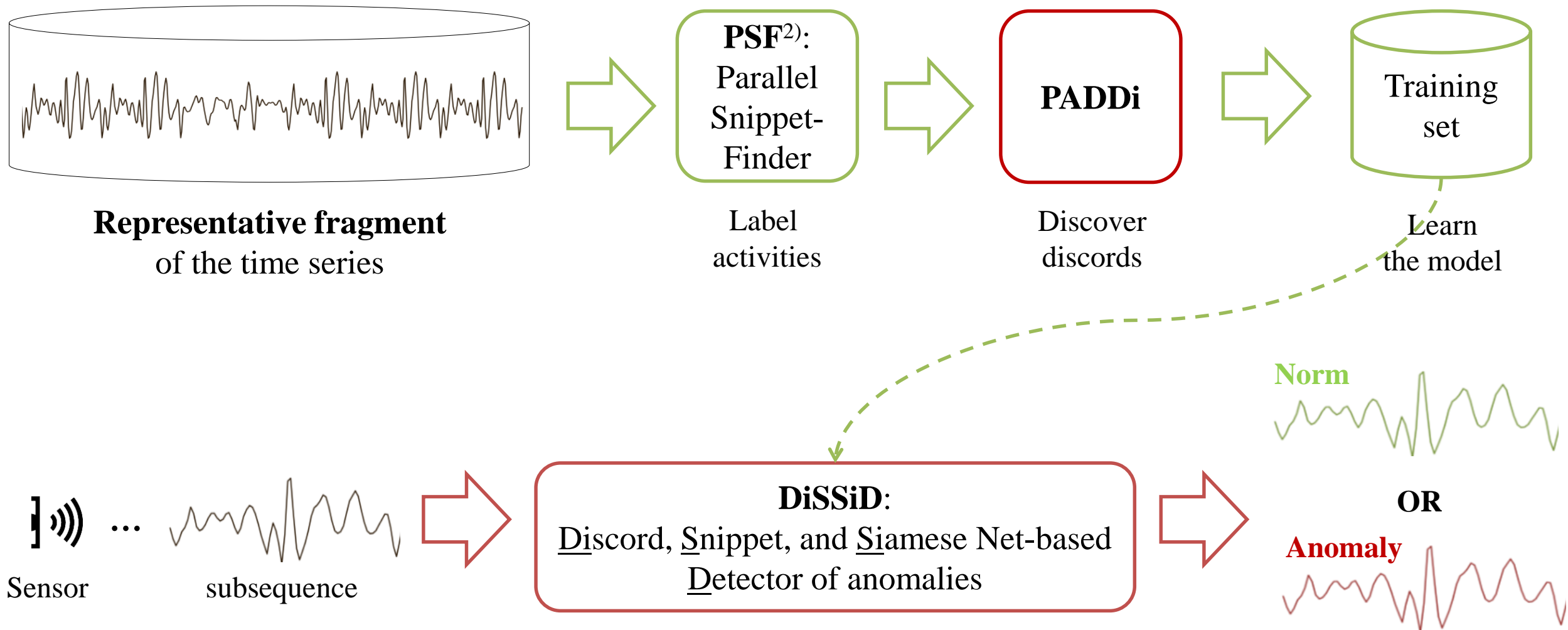


Top-6: $m = 187$ (3 min 7 sec)



* Weng Q. *et al.* MLaaS in the wild: workload analysis and scheduling in large-scale heterogeneous GPU clusters. NSDI 2022. pp. 945-960. <https://www.usenix.org/conference/nsdi22/presentation/weng>

Q: Can we discover anomalies online? A: DiSSiD¹⁾



¹⁾ Kraeva Ya.A. Detection of time series anomalies based on data mining and neural network technologies. Bulletin of SUSU, CMSE. 2023. 12(3). pp. 50-71. DOI: [10.14529/cmse230304](https://doi.org/10.14529/cmse230304).

²⁾ Zymbler M., Goglachev A. Fast summarization of long time series with graphics processor. Mathematics. 2022. 10(10). 1781. DOI: [10.3390/math10101781](https://doi.org/10.3390/math10101781).

We're ready to discover anomalies in your time series ASAP!

- **PADDi (recently completed research)**
The first, only, and scalable algorithm for all-length discord discovery in long time series on multi-GPU clusters
- **DiSSiD (ongoing research)**
Deep learning method based on PADDi for *online anomaly discovery in time series*
- **Further research**
Extension of the proposed solutions to *multidimensional time series*



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